Time-Dependent Reliability Analysis of Large Systems under Non-Gaussian Loading

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Outline

- **Background**
- **Response of a Linear System under Gaussian Loading**
- **Outline of our Approach**
- **Random Vibrations of Non-linear Systems under Non-Gaussian Loading**
  - ❖ **Review of PCE-KL-QMC Method**
  - ❖ **Metamodel of Autocorrelation Approach**
  - ❖ **Autocorrelation of Linearized System**
  - ❖ **Autoregressive (AR) Extrapolation to address burn-in period**
  - ❖ **Automotive Truck Example**
- **Conclusions**

Background

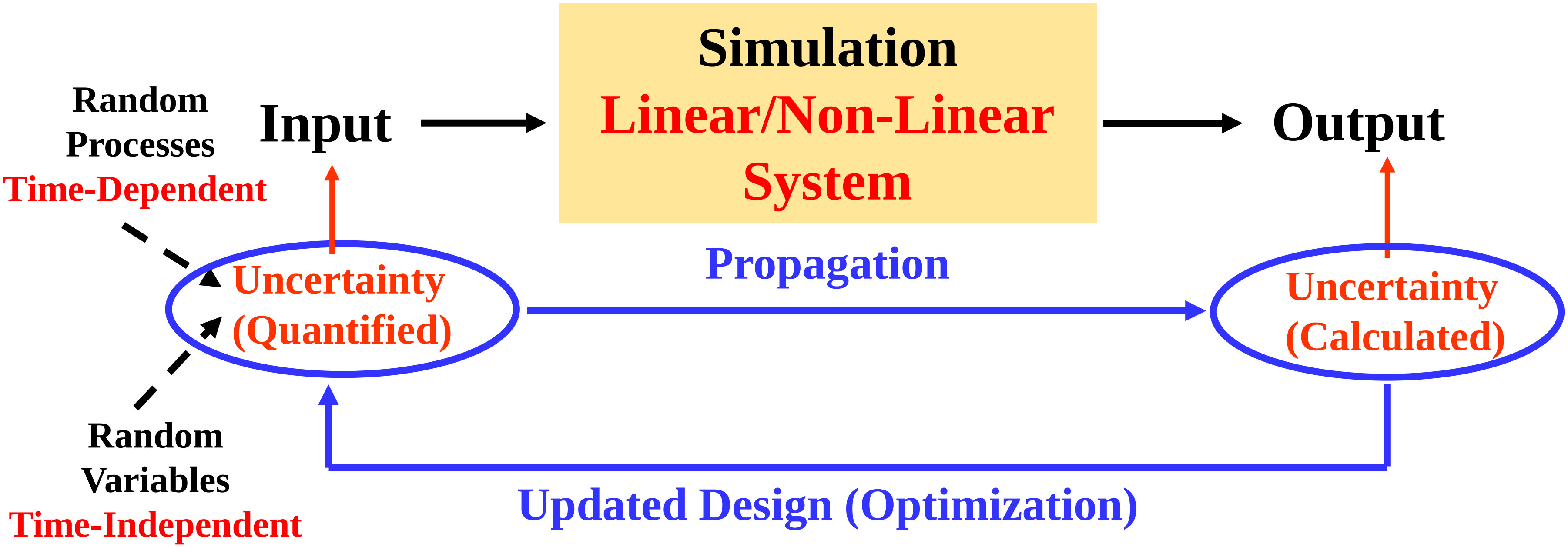
The response of time-dependent systems is a **random process**

- **Time-dependent reliability** considers **performance through time**. Such a design can, among others:
  - ✓ Reduce warranty cost
  - ✓ Increase customer satisfaction
  - ✓ Identify maintenance schedules

- **Real systems** are **large with millions of degrees of freedom DOF (> 5 million)**

- Small number of **nonlinear components** (e.g. active suspension, tires) exist in **large vibratory systems** along with **large linear vibratory subsystems** (e.g. trim body).

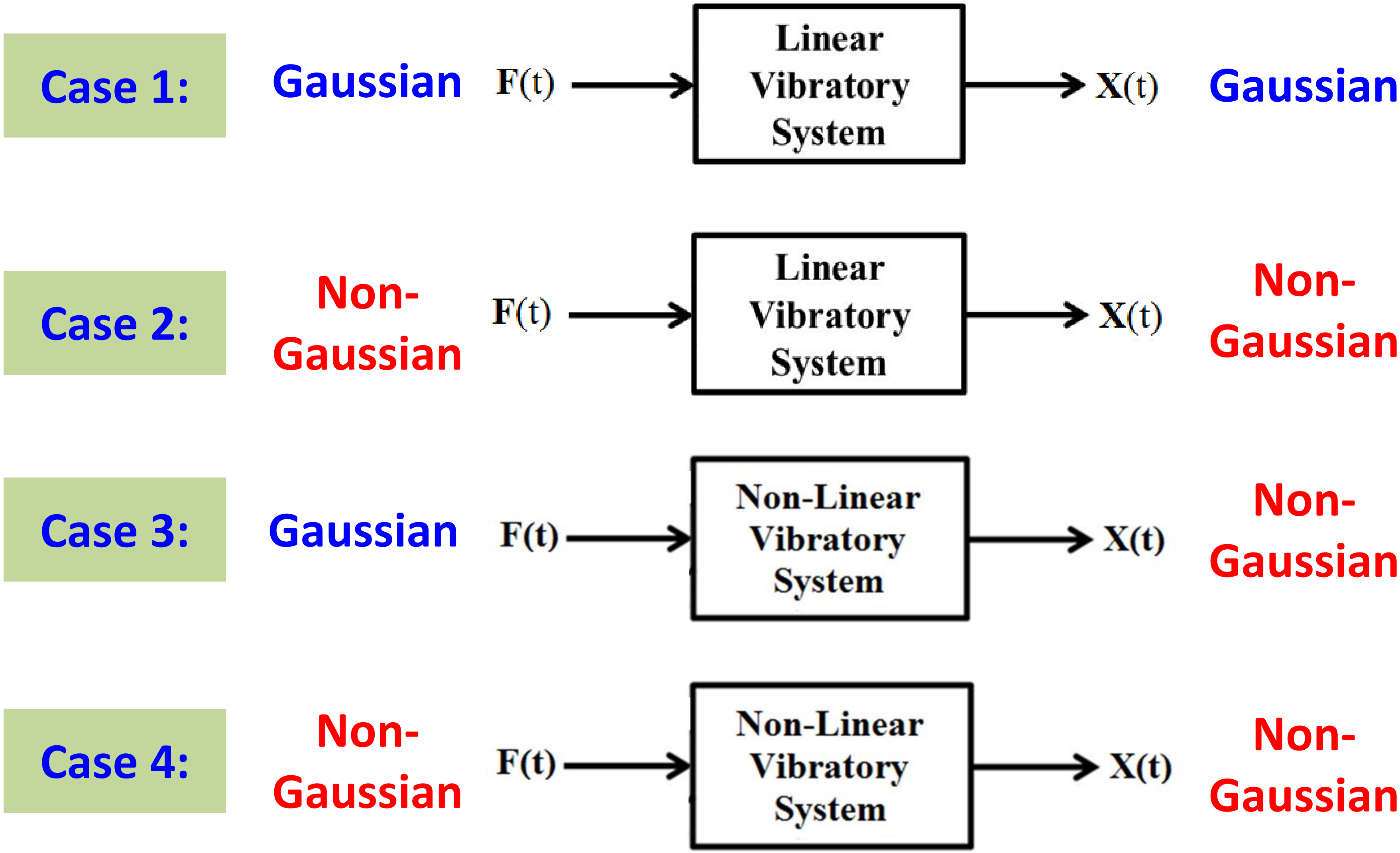
Design Under Uncertainty



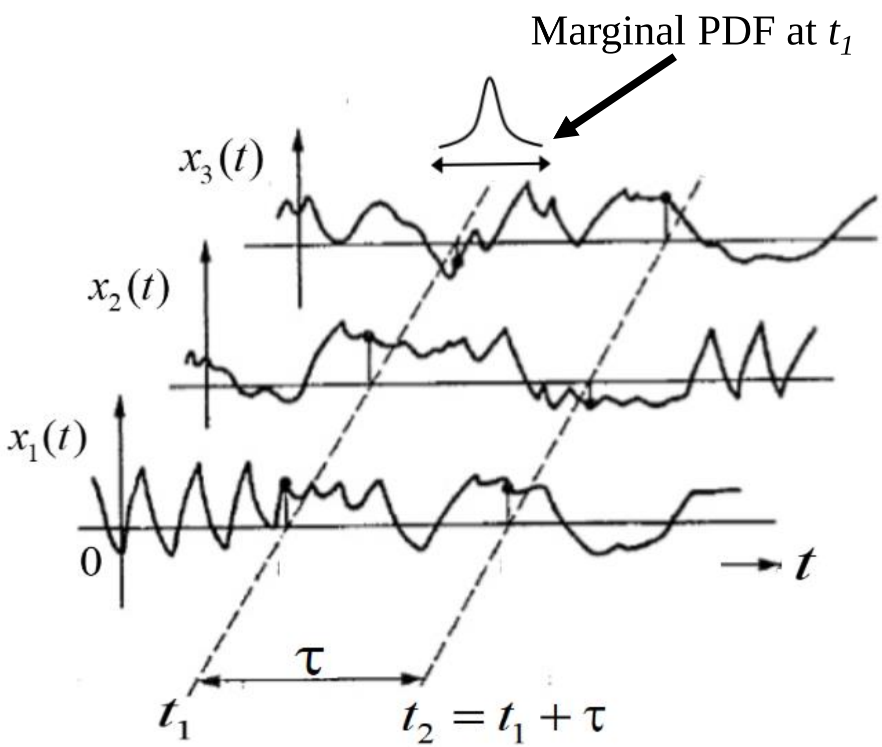
Challenges:

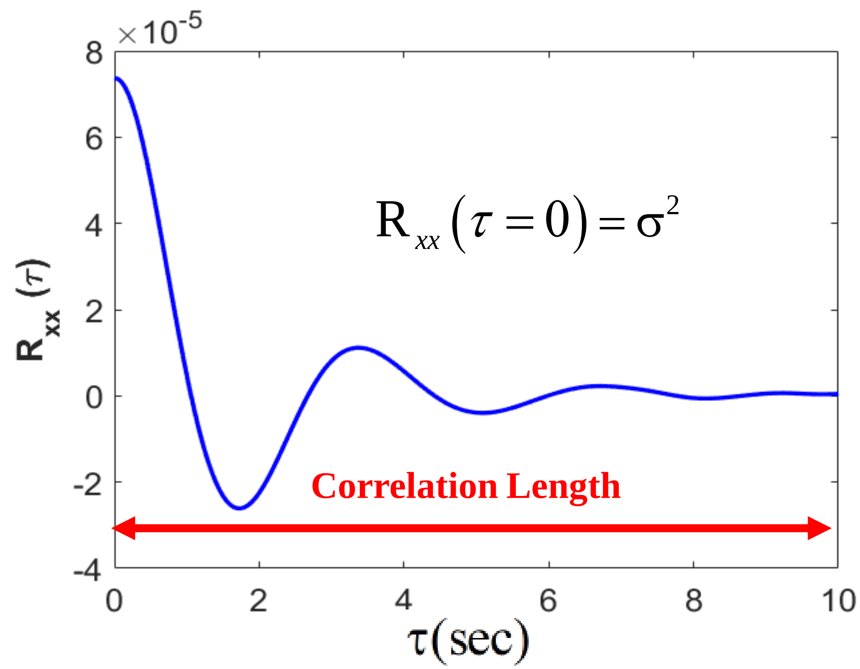
- Quantification of a Input Random Process **(Gaussian/Non-Gaussian)**.
- Calculation of Output Uncertainty **(Gaussian/Non-Gaussian)**.
- Propagation of Uncertainty **(Linear/nonlinear Systems)**.
  - ❖ Reduce the **number of system simulations needed** for TD-RBDO.
  - ❖ Reduce computational **cost of each simulation** for large vibratory systems.

Quantification and Propagation of Uncertainty



Quantification of a Random Process





A zero mean, stationary Gaussian process is fully characterized by its autocorrelation function.

For a non-Gaussian process we need skewness and kurtosis in addition to the autocorrelation function.

Response of a Linear Vibratory System

Stationary, Gaussian Input Random Processes → $\mathbf{F}(t)$

Input Random Variables (e.g. Stiffness, Damping) → $\mathbf{X}$

Linear Vibratory System

→ $\mathbf{Y}(t)$ → **Output Processes** (Stationary, Gaussian)

PSD of Output → $$S_{YY}(\omega) = |H(\omega)|^2 S_{FF}(\omega)$$ ← **PSD of Input**

FRF

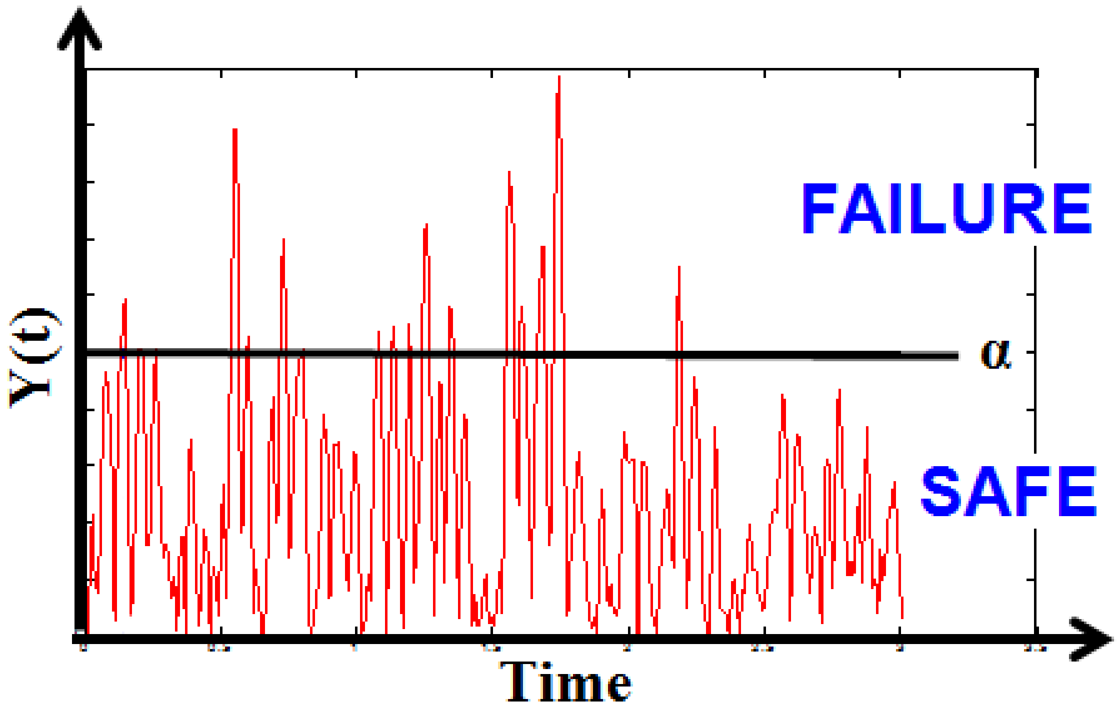
$$P_f(0,T) = P_f^0 + \left(1 - P_f^0\right)\int_0^T f(t)\,dt$$

Time-Dependent Probability of Failure

$P_f^0 = P\{Y(0) > \alpha\}$: Instantaneous probability of failure at t=0.

$f(t)$: PDF of the first time to failure calculated using notion of up-crossing (v^+) and joint up-crossing (v^{++}) rate.

Definition of Failure



Failure if defined as an event where, response exceeds certain threshold.

Schematic of TD-RBDO Approach (Simulation-Based)

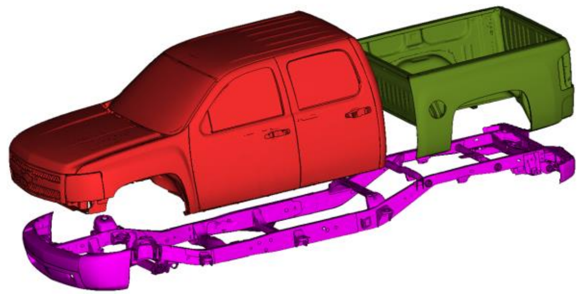
Non-Gaussian Loading and Non-Linear System

Random Variables

Example: m_1, k_1, etc

Random Processes

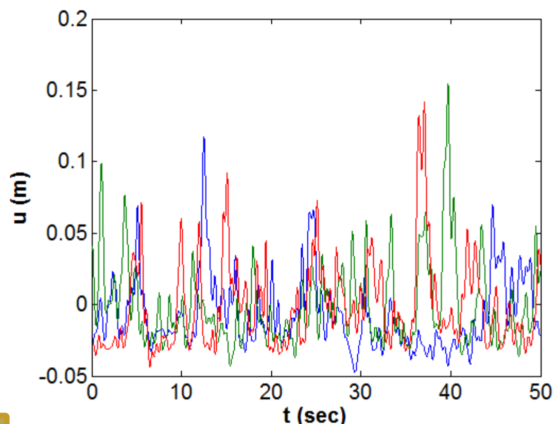
e.g. Rough Road



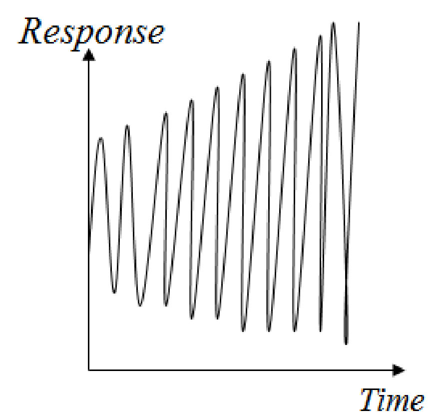
Response

Time Dependent Reliability Analysis

$$[m(\mathbf{X})]\ddot{\mathbf{Y}}(t)+[c(\mathbf{X})]\dot{\mathbf{Y}}(t)+[k(\mathbf{X})]\mathbf{Y}(t)+\mathbf{h}(\mathbf{Y},\dot{\mathbf{Y}},\ddot{\mathbf{Y}},\mathbf{X})=\mathbf{F}(t)$$



Component Mode Synthesis (CMS) after partitioning the system into linear and nonlinear substructures for efficient system simulation.



Metamodel of Output Autocorrelation Function for Random Vibration of Non-Linear Systems

Background: PCE-KL Method

"Characterization" of random process → Development of a **stochastic metamodel** for generating **trajectories** of the process

Polynomial Chaos Expansion – Karhunen Loeve Expansion (PCE-KL)

$$Z(t) = \sum_{i=0} b_i(t)\Psi_i(t) = b_0(t) + b_1(t)\xi(t) + b_2(t)\left(\xi^2(t) - 1\right) + b_3(t)\left(\xi^3(t) - 3\xi(t)\right) + b_4(t)\left(\xi^4(t) - 6\xi^2(t) + 3\right) + \cdots$$

b_i: coefficients to be calculated

$\xi(t)$: Standard Normal Process

Define the first 4 "target" moments of non-Gaussian random variable Z

$$E[Z] = \mu_Z = b_0$$

$$m_Z^2 = E[(Z - b_0)^2] = \ldots$$

$$m_Z^3 = E[(Z - b_0)^3] = \ldots$$

$$m_Z^4 = E[(Z - b_0)^4] = \ldots$$

4 Equations ,
4 Unknowns (b_0, b_1, b_2, b_3)

↓

Solve optimization problem

Background: PCE-KL Method

Using orthogonality properties of Hermite polynomials:

$$C_{ZZ}(t_1,t_2) = \sum_{i=1} b_i(t_1) b_i(t_2) \cdot (i!) \cdot \left(E[\xi(t_1)\xi(t_2)]\right)^i$$

$C_{ZZ}(t_1,t_2)$: Covariance

Known (given) — $C_{ZZ}(t_1,t_2)$

Only Unknown (Calculate) — $E[\xi(t_1)\xi(t_2)]$

$$C_{\xi\xi}(t_1,t_2) = E[\xi(t_1)\xi(t_2)]$$ ⇒ **Trajectories of ξ(t) using K-L Expansion**

K-L Expansion

$$\xi(t) = \sum_{i=1}^{N} \sqrt{\lambda_i} \cdot f_i(t) \cdot \xi_i$$

λ_i: Eigenvalues of $C_{\xi\xi}(t_1,t_2)$

$f_i(t)$: Eigenvectors of $C_{\xi\xi}(t_1,t_2)$

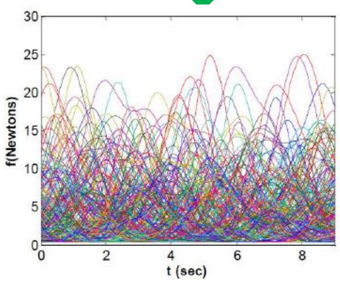
ξ_i: Independent standard normal variables

Realization of ξ_i in **N dimensions**

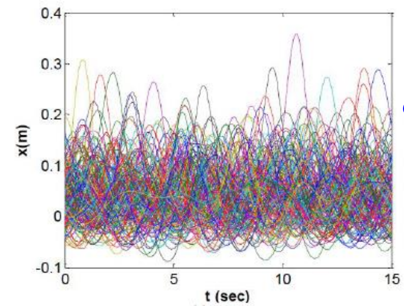
Background: PCE-KL-QMC Method

M "space filled" trajectories in N dimensions

Quasi Monte Carlo Method (QMC)



Nonlinear Vibratory System



"M" Output trajectories

Moments and Autocorrelation of output

M is the number of system simulations required.

As N increases, M increases considerably.

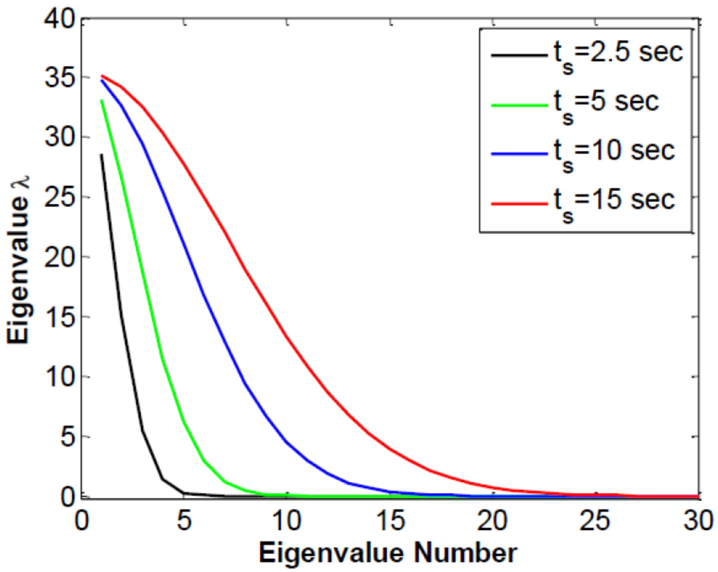
Generate new output trajectories without solving the system

Computational Cost of Quasi Monte Carlo Method

N (no. of dimensions in KL) = no. of significant eigenvalues

$$t_S \geq \max\left(\tau_{F_1}^{cor}, \cdots, \tau_{F_n}^{cor}, \tau_{Y_1}^{cor}, \ldots, \tau_{Y_m}^{cor}\right)$$

t_s - Dominated by correlation length of output



For a narrow-band process, the correlation length is large e.g. (> 15 sec).

Simulation Time (s)	No. of Eigenvalues in KL	No. of simulation of large vibratory system
15	30	>500
9	13	200
3	6	60

Computational cost increases considerably with increasing simulation length

Metamodel Approach for Autocorrelation Function

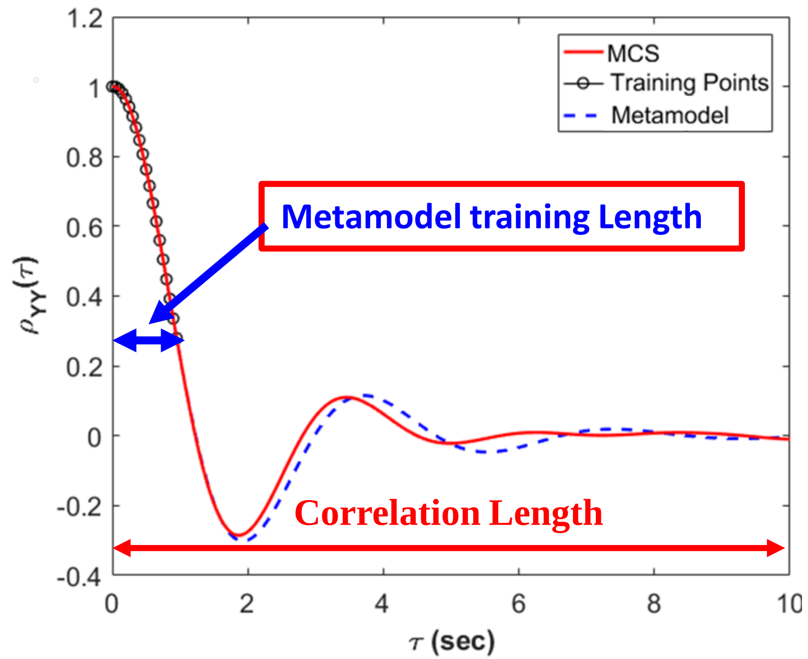
- A set of decaying sinusoids is used to approximate the autocorrelation function (inverse Fourier transform of a Gaussian Function).
- A global Genetic Algorithm fits all parameters.

$$R_{yy}(\tau) = \sum_{k=1}^{n} A_k e^{-\gamma_k \tau} \sin(\omega_k - \alpha_k)$$

$\gamma =$ decay constant $\alpha =$ phase

$A =$ amplitude $\omega =$ frequency

$n =$ number of terms in summation



- ✓ **Small number** of system simulations required.
- ✓ **Duration** of each simulation is short.

No. of Terms - *n* and Optimal Training Length

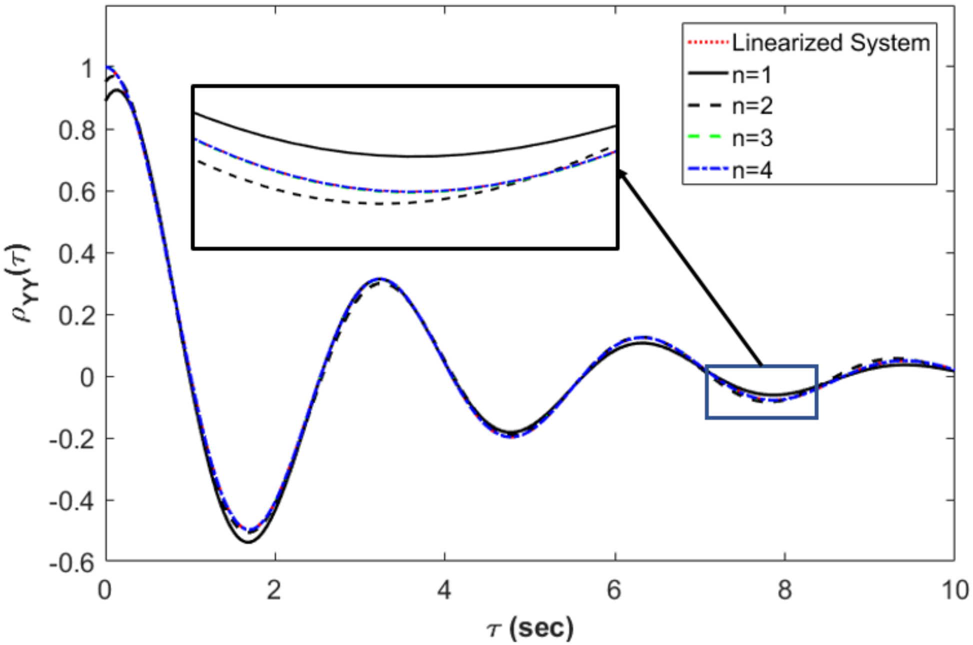
Develop an initial "fit" using the autocorrelation of the "linearized" System:

$$S_{YY}(\omega) = |H(\omega)|^2 S_{FF}(\omega)$$

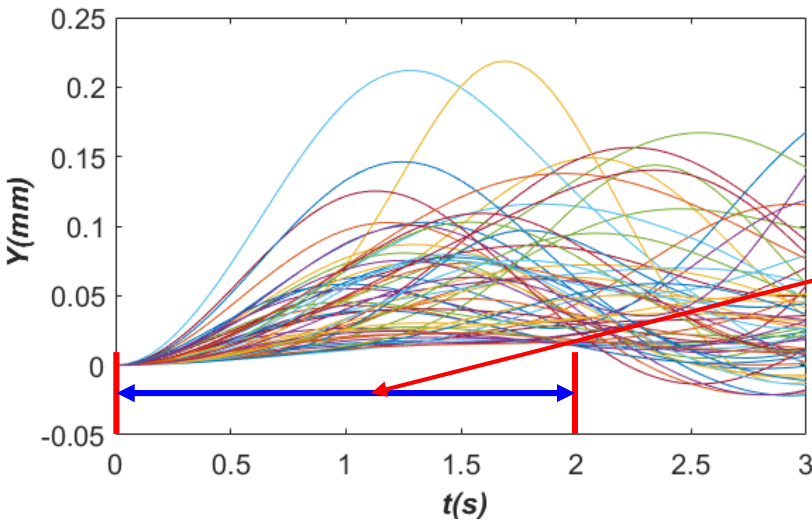
FRF of Linearized System (e.g. linearized at mean)

Inverse Fourier of PSD $S_{YY}(\omega)$ provides the **autocorrelation** of the **linearized system**.

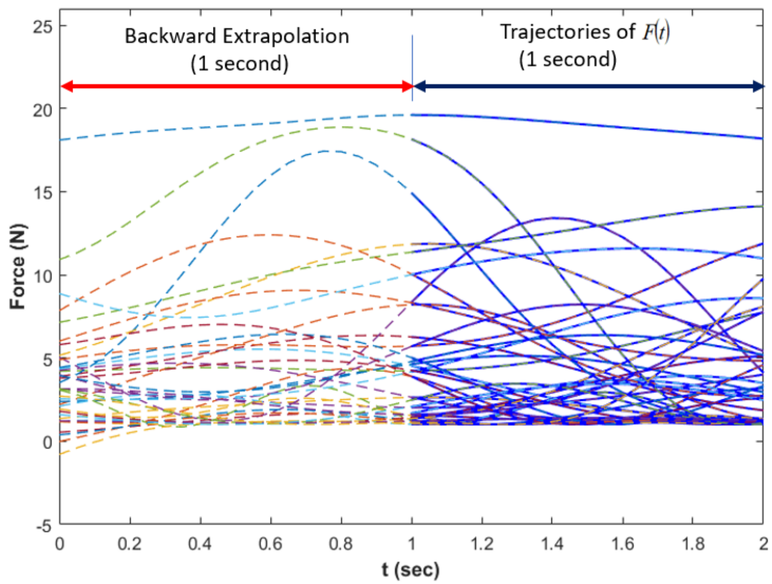
n	Root-Mean-Square Error (RMSE)
1	0.0191
2	0.0103
3	0.0003581
4	0.0003479



AR Extrapolation to Address Burn-in period



Transient but artificial effect of initial conditions



Auto-regressive (AR) Extrapolation using Burg's Method

$$y_n = -\sum_{m=1}^{p} a_m y_{n-m} + e_n$$

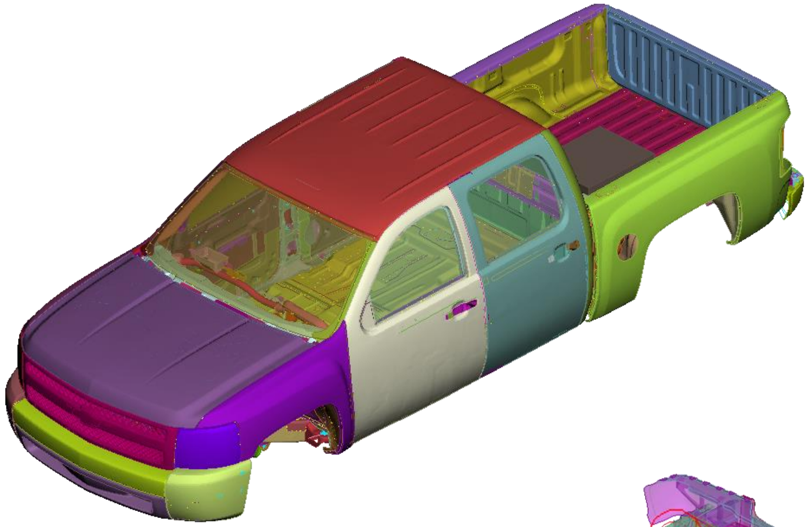
y_n = samples of time series

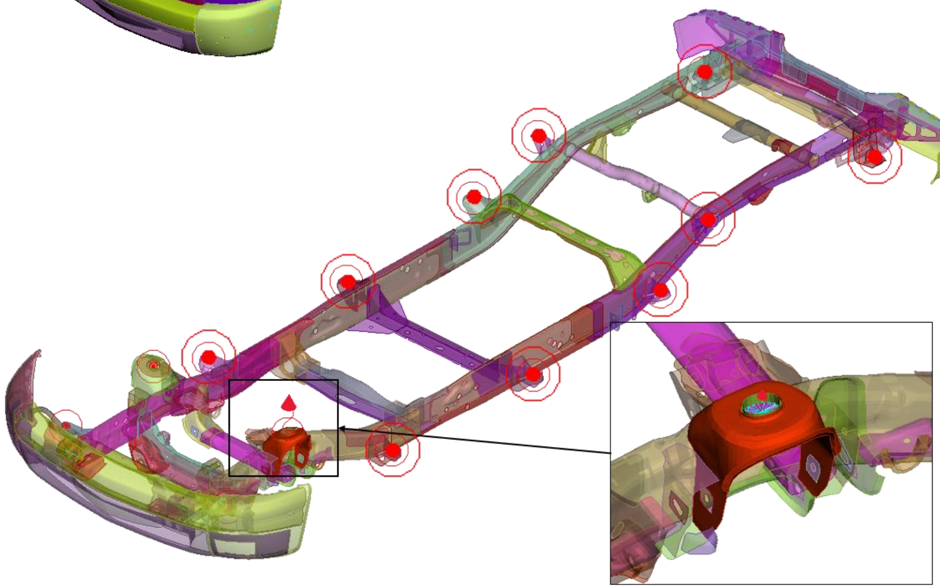
a_m = model coefficients

p = order of the model

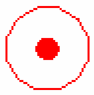
e_n = residual

Automotive Truck with Non-Linear Mounts





4.5 Million DOFs



Non-linear mount locations

Output Process $y(t)$

Non-Gaussian displacement at seat attachment point

Input Process $F(t)$

Non-Gaussian force

Characterization of Input Process F(t)

Weibull distribution

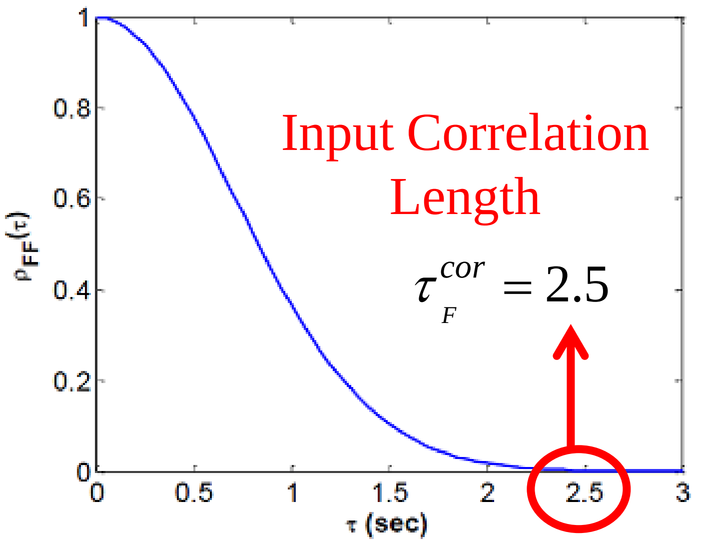
Parameters: scale=5 , shape=1.2

First 4 statistical moments

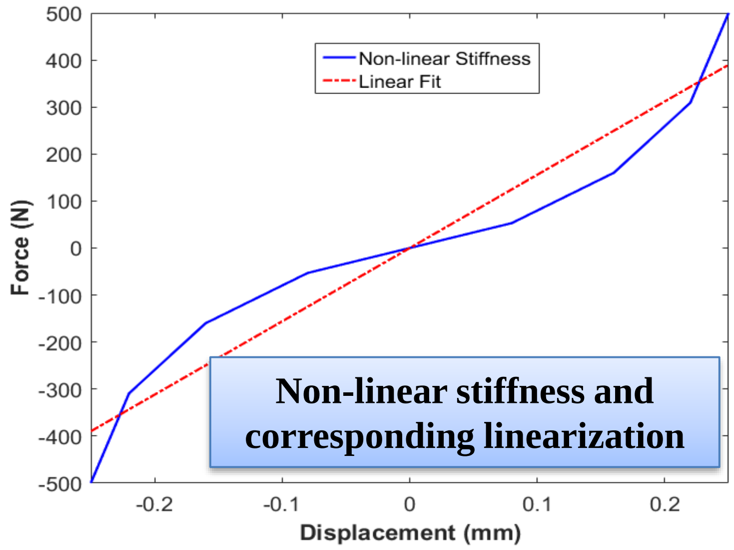
Quantity	Description	Value
μ_1	Mean	4.7
$m_2 (= \sigma^2)$	Variance	15.5
m_3 / σ^3	Skewness Coefficient	1.5
m_4 / σ^4	Kurtosis Coefficient	6.2

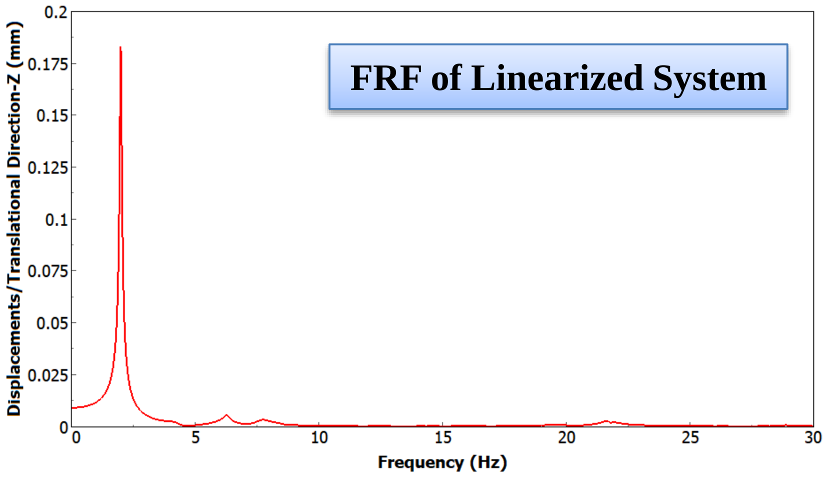
Create PCE stochastic metamodel for input process and generate trajectories of F(t).

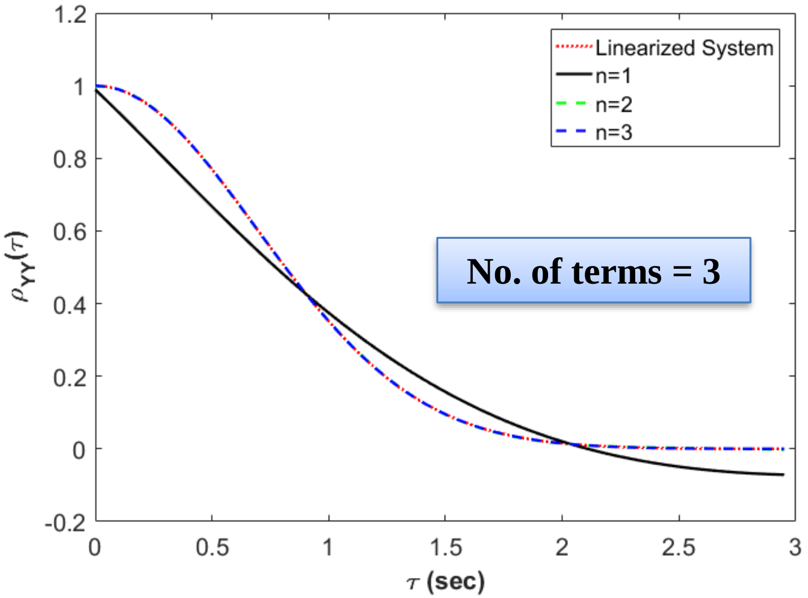
Autocorrelation Function

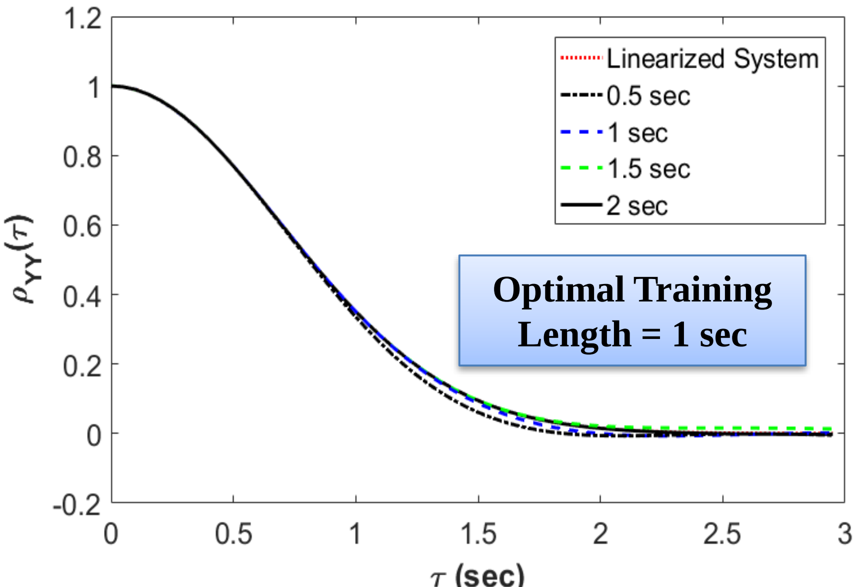


Characterization of Input Process F(t)







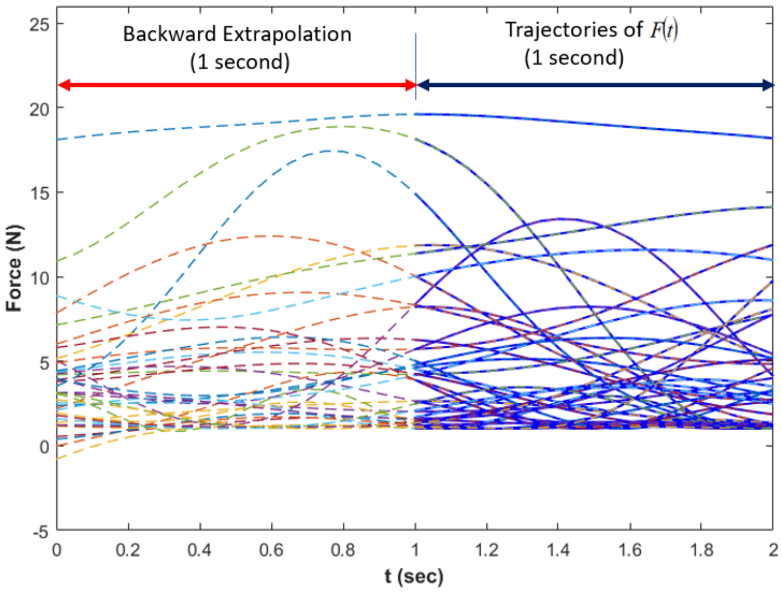


Characterization of Input Process F(t)

- For training length of 1 sec, 3 dominant eigenvalues are needed for KL expansion of input process.
- We use a space filling design (OSLH) in 3 dimensions to get M space-filled trajectories of input process.
- Burn in Period for this example is 1 seconds. Backward AR Extrapolation for additional 1 second is performed to address that

QMC efficiency for moment estimation

# of Sample Pts M	Mean (N)	Std. Dev. (N)	Skewness	Kurtosis
10	4.45	3.8	0.87	2.33
20	4.53	3.61	0.85	2.58
30	4.68	3.53	1.05	3.46
40	4.72	4.19	1.59	5.35
250,000 MCS	4.70	3.94	1.52	6.15
Target	4.70	3.94	1.52	6.24



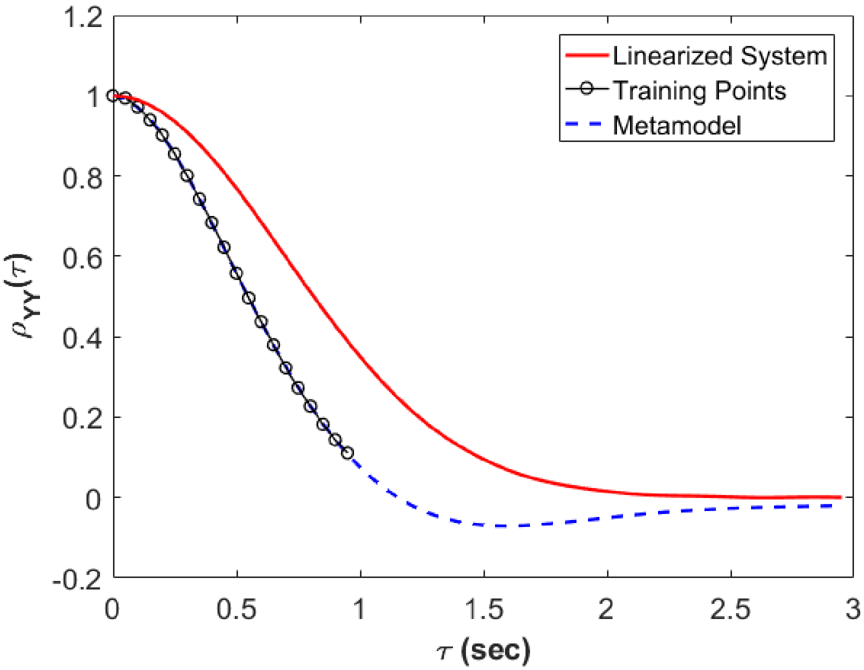
Characterization of Output Process Y(t)

- Calculate the 4 moments and partial autocorrelation of output displacement process y(t).
- Develop the metamodel of autocorrelation function.

Moments of Output

# of Sample Pts M	Mean (N)	Std. Dev. (N)	Skewness	Kurtosis
10	0.035	0.027	0.70	2.06
20	0.039	0.0296	0.7	2.07
30	0.0373	0.0305	1.0	3.17
40	0.0374	0.0311	1.50	5.19
50	0.0374	0.0310	1.48	5.20

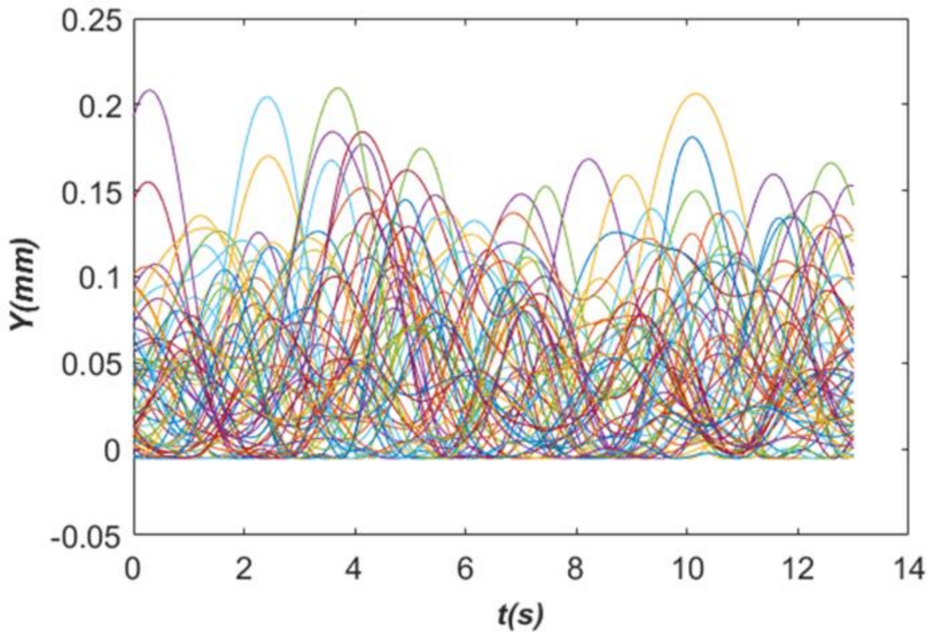
Term No.	Amplitude (N)	Decay constant (N)	Frequency	Phase
1	0.3922	2.4502	373.7885	8.4962
2	0.7091	1.7275	126.8746	5.1885
3	0.7044	1.8044	374.8402	4.4764



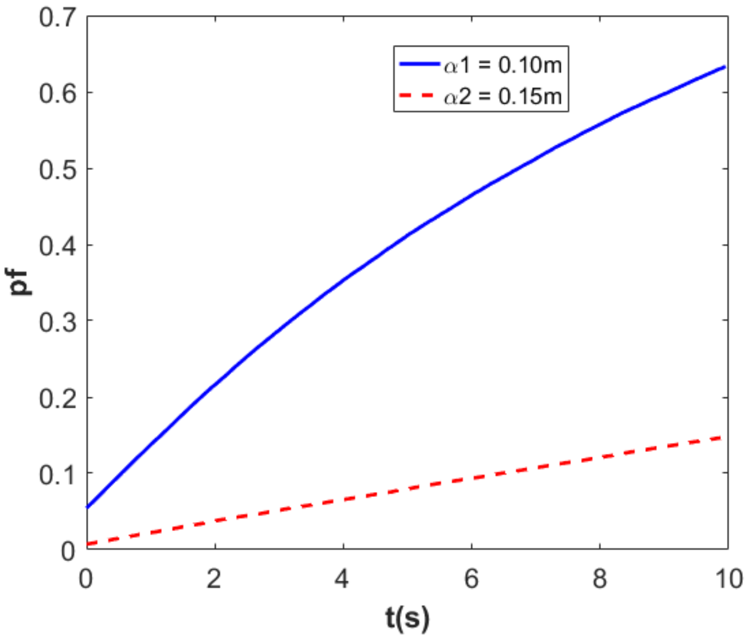
Parameters of Metamodel of Output Autocorrelation Function

Time-Dependent Probability of Failure

- Develop **PCE stochastic metamodel** for output process Y(t).
- Generate **new trajectories** of Y(t), using the metamodel to calculate time-dependent probability of failure.

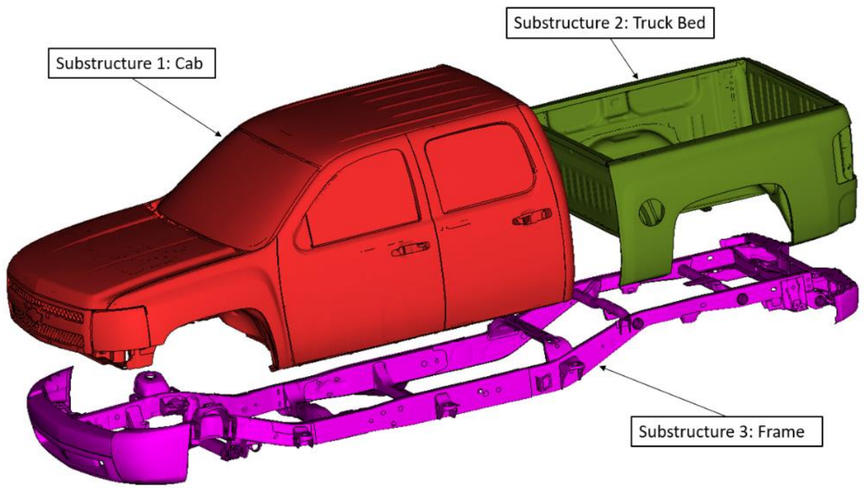


Trajectories of Output



Probability of Failure

Craig-Bampton CMS and Total Cost



No	Substructure	Modal DOFs	Physical DOFs	Total DOFs
1	Cab	163	-	-
2	Bed	64	-	-
3	Frame	103	-	-
0	Residual	-	6	336

4.5 million DOF -> to **336 modal** DOFs

Non-linear transient simulations needed for **time-integration**

Number of simulations required = 40
Duration of each simulation = 2 sec.

EOM for *i*th Substructure

$$\begin{bmatrix} \mathbf{m}_i^{\Gamma\Gamma} & \mathbf{m}_i^{\Gamma\Omega} \\ \mathbf{m}_i^{\Omega\Gamma} & \mathbf{m}_i^{\Omega\Omega} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_i^{\Gamma} \\ \ddot{\mathbf{u}}_i^{\Omega} \end{Bmatrix} + \begin{bmatrix} \mathbf{k}_i^{\Gamma\Gamma} & \mathbf{k}_i^{\Gamma\Omega} \\ \mathbf{k}_i^{\Omega\Gamma} & \mathbf{k}_i^{\Omega\Omega} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_i^{\Gamma} \\ \mathbf{u}_i^{\Omega} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_i^{\Gamma} \\ \mathbf{f}_i^{\Omega} \end{Bmatrix}$$

Craig-Bampton transformation using substructure normal modes and constraint modes)

No.	Substructure	Computational Cost
1	Cab Modal Model	14 min x1
2	Bed Modal Model	2 min x 1
3	Frame Model	3 min x 1
4	Residual Time Integration	**2 sec. x 40 = 80 sec.**

Summary and Conclusions

- Presented a method for time-dependent reliability analysis of **non-linear systems** under **non-gaussian loads using a** Metamodel of Autocorrelation approach.
- This approach drastically reduces number of system simulations required.
- **AR Extrapolation** to address the effect of **Burn-in period**.
- **CMS** to handle **non-linear components.**

Example demonstrated the accuracy and efficiency of above techniques.

Questions & Answers

THANK YOU