AN EFFICIENT RE-ANALYSIS METHODOLOGY FOR OPTIMIZATION OF LARGE-SCALE VIBRATION SYSTEMS

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ABSTRACT - Finite-element (FE) analysis is a well-established methodology in structural dynamics. However, optimization and/or probabilistic studies can be prohibitively expensive because they require repeated FE analyses of large models. Various re-analysis methods have been proposed with the premise to effectively calculate the dynamic response of a structure after a baseline design has been modified, without recalculating the new response.

The parametric reduced-order modelling (PROM) and the combined approximations (CA) are two re-analysis methods, which can handle large model parameter changes in a relatively efficient manner. Although both methods are promising by themselves, they can not handle large FE models with a large number of degrees of freedom (DOF) (e.g. greater than 100,000) and a large number of design parameters (e.g. greater than 50), which are common in practice. In this paper, the advantages and disadvantages of the PROM and CA methods are first discussed in detail.

Subsequently, a new re-analysis method is proposed where the original CA method is modified to further improve its efficiency, especially for problems where a large number of modes must be retained. The modified CA (MCA) method and Kriging interpolation are then integrated with the PROM approach, to formulate an efficient re-analysis method which can be used for optimization studies of complex structures. The method can efficiently handle large FE models with many design parameters that vary in a wide range.

A vibro-acoustic analysis of a realistic vehicle finite-element model is presented to demonstrate the efficiency and accuracy of the new re-analysis method and a design optimization study highlights its capabilities.

An Efficient Re-Analysis Methodology for Optimization of Large-Scale Vibration Systems

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Overview

- Introductory Remarks
  - Reduced-Order Modeling in Structural Dynamics
- Review of Existing Re-analysis Methods
  - Parametric Reduced-Order Modeling (PROM) Method
  - Combined Approximation (CA) Method
- New Re-Analysis Method
  - Modified Combined Approximation (MCA) Method
  - Integration of MCA with Optimization
- Summary and Conclusions

Reduced-Order Modeling (ROM) (Modal Model)

- Eigenvalue Analysis
  - Lanczos Algorithm
- "Math" Substructuring
  - Automated Multi-Level Substructuring (AMLS)
- "Physical" Substructuring
  - Component Mode Synthesis (CMS)
    - Craig-Bampton (Fixed Interface)
    - CMS with Interface Modes

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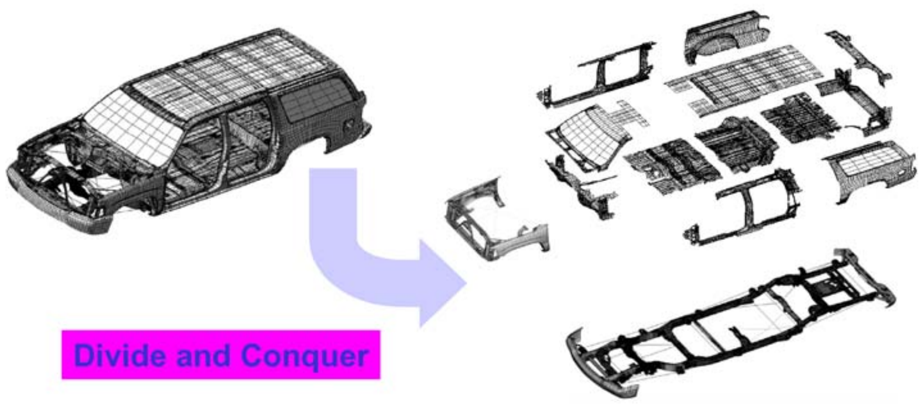
Reduced-Order Modeling



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Reduced-Order Modeling

$$\left[\mathbf{K} - \omega^2 \mathbf{M}\right]\mathbf{d} = \mathbf{F}$$

Modal Representation: $\mathbf{d} = \mathbf{\Phi U}$

Modal Basis: $\mathbf{\Phi} = \left[\boldsymbol{\varphi}_1 \quad \boldsymbol{\varphi}_2 \quad \cdots \quad \boldsymbol{\varphi}_n\right]$

Modal Model: $\left[\mathbf{\Phi}^T\mathbf{K\Phi} - \omega^2\mathbf{\Phi}^T\mathbf{M\Phi}\right]\mathbf{U} = \mathbf{\Phi}^T\mathbf{F}$

Practical Issues:

- Basis $\mathbf{\Phi}$ must be recalculated for each new design
- Many modes must be retained
- Calculation of "triple" product $\mathbf{\Phi}^T\mathbf{K\Phi}$ can be expensive

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Our Solution

Re-analysis methods: PROM and CA / MCA

Practical Issues:

- Basis $\mathbf{\Phi}$ must be recalculated for each new design
- Many modes must be retained
- Calculation of "triple" product $\mathbf{\Phi}^T\mathbf{K\Phi}$ can be expensive

Kriging interpolation

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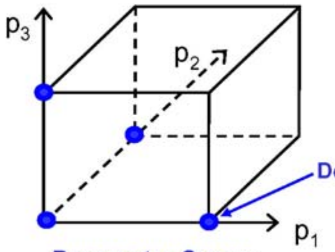
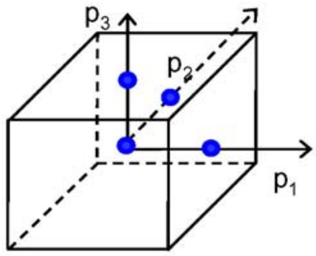
Parametric Reduced-Order Modeling (PROM)

Zhang et. al.'s Approach (Corner Points)



Balme's Approach (Center Points)



$$\mathbf{P} = [\mathbf{\Phi}_0 \quad \mathbf{\Phi}_1 \quad \mathbf{\Phi}_2 \quad \mathbf{\Phi}_3] \longleftarrow \text{Reduced Basis}$$

- **PROM method is an extension of Rayleigh-Ritz method**
- **Quality of approximation is decided by number and location of design points**
- **Both approaches require (p+1) design points**
- **Mode shapes from the first approach are less likely to be linear dependent**

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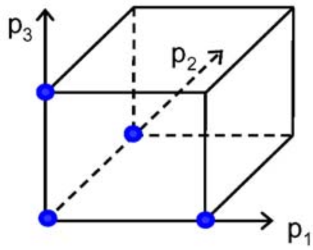
PROM: Procedure Steps



p_1,p_2,p_3: design parameters

Eigen-analysis of full matrices

$$eig(\mathbf{K}_p, \mathbf{M}_p) \Rightarrow \lambda^p, \mathbf{\Phi}_p$$

1. Form reduced basis:

$$\mathbf{P} = [\mathbf{\Phi}_0 \quad \mathbf{\Phi}_1 \quad \mathbf{\Phi}_2 \quad \mathbf{\Phi}_3]$$

2. Project system matrices to reduced basis:

$$\mathbf{K}_R = \mathbf{P}^T \mathbf{K}_p \mathbf{P} \qquad \mathbf{M}_R = \mathbf{P}^T \mathbf{M}_p \mathbf{P}$$

3. Eigen-analysis of reduced matrices:

$$eig(\mathbf{K}_R, \mathbf{M}_R) \Rightarrow \tilde{\lambda}^p, \mathbf{\Theta}$$

4. Obtain approximate eigenvectors:

$$\tilde{\mathbf{\Phi}}_p = \mathbf{P}\mathbf{\Theta}$$

- **Advantage: Re-analysis is done using the reduced basis**
- **Disadvantage: Condensation of matrices can be expensive due to large size and high density of mode matrix P**

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Combined Approximation (CA) Method

Original Eigenvalue Problem

$$\mathbf{K}_0\mathbf{\Phi}_0 = \lambda_0\mathbf{M}_0\mathbf{\Phi}_0$$

Modified Model

$$\mathbf{K}_p = \mathbf{K}_0 + \Delta\mathbf{K}$$

$$\mathbf{M}_p = \mathbf{M}_0 + \Delta\mathbf{M}$$

Direct Solution for Modified Eigenvectors

$$(\mathbf{K}_0 + \Delta\mathbf{K})\mathbf{\Phi}_p = \lambda_p\mathbf{M}_p\mathbf{\Phi}_p$$

or $$\mathbf{\Phi}_p = \lambda_p\mathbf{K}_0^{-1}\mathbf{M}_p\mathbf{\Phi}_p - \mathbf{K}_0^{-1}\Delta\mathbf{K}\mathbf{\Phi}_p$$

Iterative Solution for Modified Eigenvectors

$$\mathbf{\Phi}_{p,j} = \lambda_p\mathbf{K}_0^{-1}\mathbf{M}_p\mathbf{\Phi}_{p,j-1} - \mathbf{K}_0^{-1}\Delta\mathbf{K}\mathbf{\Phi}_{p,j-1}$$

Definition and formation of base vectors

$$\mathbf{R}_j = \mathbf{\Phi}_{p,j} - \mathbf{\Phi}_{p,j-1}$$

$$\mathbf{R}_1 = \mathbf{K}_0^{-1}\mathbf{M}_p\mathbf{\Phi}_0$$

$$\mathbf{R}_j = -\mathbf{K}_0^{-1}\Delta\mathbf{K}\mathbf{R}_{j-1} \qquad j = 2,\ldots,s$$

$$\mathbf{R} = [\mathbf{R}_1 \quad \mathbf{R}_2 \quad \cdots \quad \mathbf{R}_s]$$

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Combined Approximation (CA) Method

Project system matrices to reduced basis:

$$\mathbf{K}_R = \mathbf{R}^T\mathbf{K}_p\mathbf{R} \qquad \mathbf{M}_R = \mathbf{R}^T\mathbf{M}_p\mathbf{R}$$

Eigen-analysis of reduced matrices:

$$eig(\mathbf{K}_R, \mathbf{M}_R) \Rightarrow \tilde{\lambda}^p, \mathbf{\Theta}$$

Obtain approximate eigenvectors:

$$\tilde{\mathbf{\Phi}}_p = \mathbf{R}\mathbf{\Theta}$$

- **Advantages:**
  1. Very accurate because base vectors are updated at every new design point
  2. Decomposition is performed once on K_0
- **Disadvantages:**
  1. For a large number of simulations, the cost to obtain base vectors R may be prohibitive
  2. Savings from decomposing only K_0 may be negligible for a large number of modes

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CA Method: Cost vs. Number of Modes

Major cost of CA method

$\mathbf{R}_1 = \mathbf{K}_0^{-1}\mathbf{M}_p\mathbf{\Phi}_0$ ⟷ SOLVE = DCMP + FBS

- **Example Problem:**
  - Model: Vehicle with 65,000 DOF
  - Machine: SUN ULTRA workstation
  - Software: Nastran V2001

	1 Mode	500 Modes
DCMP Cost	4 seconds	4 seconds
FBS Cost	0.1 second	50 seconds
Total Cost of SOLVE	4.1 seconds	54 seconds
Conclusion	**Cost dominated by DCMP**	**Cost dominated by FBS**

- **When cost of SOLVE is dominated by FBS, the computational advantage from inverting K_0 only once is insignificant**

Modified Combined Approximation (MCA) Method

Direct Solution of Modified Eigenproblem

$$\mathbf{K}_p\mathbf{\Phi}_p = \lambda_p\mathbf{M}_p\mathbf{\Phi}_p$$

or $$\mathbf{\Phi}_p = \lambda_p\mathbf{K}_p^{-1}\mathbf{M}_p\mathbf{\Phi}_p$$

Iterative Solution of Modified Eigenproblem

$$\mathbf{\Phi}_{p,j} = \lambda_p\mathbf{K}_p^{-1}\mathbf{M}_p\mathbf{\Phi}_{p,j-1}$$

Definition and formation of basis vectors

$$\mathbf{T}_1 = \mathbf{K}_p^{-1}\left(\mathbf{M}_p\mathbf{\Phi}_0\right)$$
$$\mathbf{T}_i = \mathbf{K}_p^{-1}(\mathbf{M}_p\mathbf{T}_{i-1}) \qquad i = 2,3,\cdots,s$$
$$\mathbf{T} = \left[\mathbf{\Phi}_0 \quad \mathbf{T}_1 \quad \mathbf{T}_2 \quad \cdots \quad \mathbf{T}_s\right]$$

CA vs. MCA

CA Method

$$\mathbf{R}_1 = \mathbf{K}_0^{-1}\mathbf{M}_p\mathbf{\Phi}_0$$
$$\mathbf{R}_j = -\mathbf{K}_0^{-1}\Delta\mathbf{K}\mathbf{R}_{j-1} \quad j = 2,\ldots,s$$
$$\mathbf{R} = \begin{bmatrix}\mathbf{R}_1 & \mathbf{R}_2 & \cdots & \mathbf{R}_s\end{bmatrix}$$

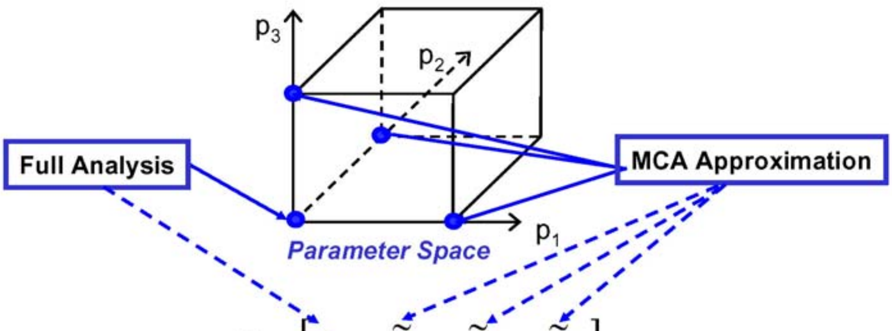
$\longleftrightarrow$

MCA Method

$$\mathbf{T}_1 = \mathbf{K}_p^{-1}\left(\mathbf{M}_p\mathbf{\Phi}_0\right)$$
$$\mathbf{T}_i = \mathbf{K}_p^{-1}(\mathbf{M}_p\mathbf{T}_{i-1}) \qquad i = 2,3,\cdots,s$$
$$\mathbf{T} = \begin{bmatrix}\mathbf{\Phi}_0 & \mathbf{T}_1 & \mathbf{T}_2 & \cdots & \mathbf{T}_s\end{bmatrix}$$

- **Comparison between CA and MCA**
  - Both have high accuracy, due to updated basis vectors at each new design
  - MCA spends more effort on DCMP of K_p
  - MCA needs fewer iterations (smaller *s*)
  - MCA uses a smaller basis (T vs. R)
  - Overall cost of MCA is less than CA if a large number of modes is retained

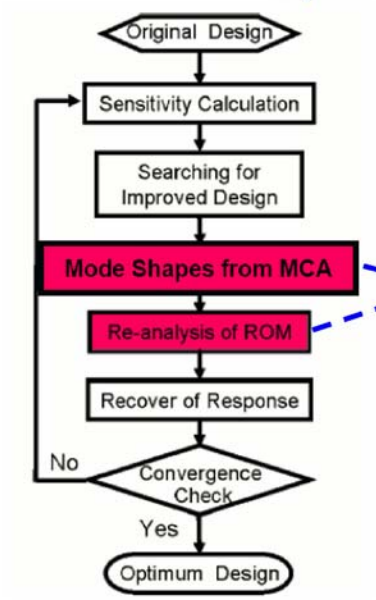
Integration of MCA and PROM Methods



$$\mathbf{P} = \begin{bmatrix}\mathbf{\Phi}_0 & \tilde{\mathbf{\Phi}}_1 & \tilde{\mathbf{\Phi}}_2 & \tilde{\mathbf{\Phi}}_3\end{bmatrix}$$

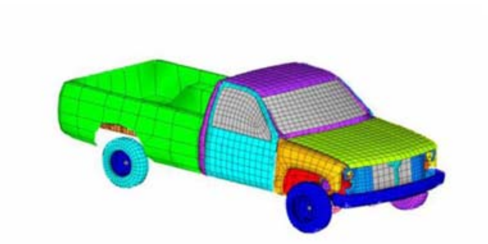
- Beneficial for very large number of re-analysis (Probabilistic studies, Genetic Algorithms)

Integration of MCA and Gradient-Based Optimization



Advantage:
MCA reduces cost to calculate new mode shapes needed in Nastran modal analysis

Numerical Example: Vehicle Model



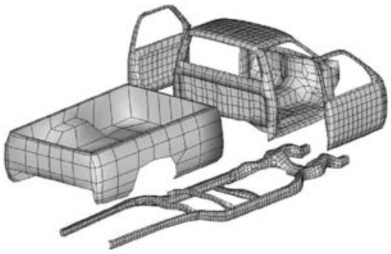
Model:
Pickup truck with 65,000 DOF

Excitation:
Unit harmonic force applied at engine mount points in X, Y and Z directions

- **Hardware and Software**:
  - Machine: SUN ULTRA workstation
  - Software: Nastran V2001

Response:
Sound pressure level (SPL) at driver's ear location

Vehicle Model: Design Change

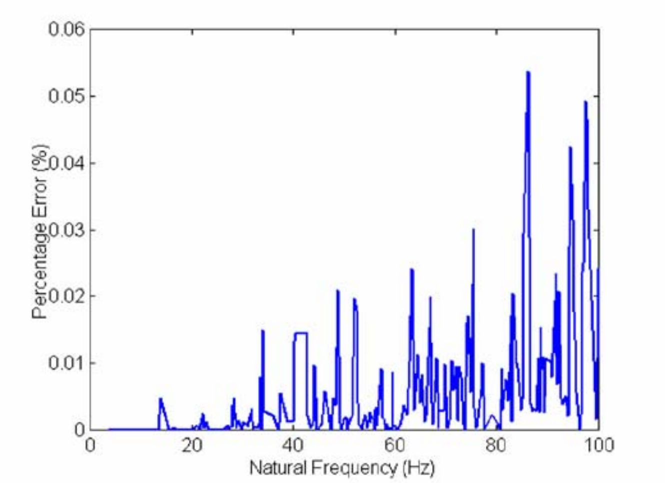


Param. #	Description (thickness of)	Initial Design	Modified Design
1	Chassis	3.137 mm	6.274 mm
2	Chassis cross link	3.611 mm	7.222 mm
3	Cabin	2.370 mm	4.740 mm
4	Bed	2.500 mm	5.000 mm
5	Left and right doors	1.240 mm	2.480 mm

- Large parameter changes (100%)

Accuracy of Modes

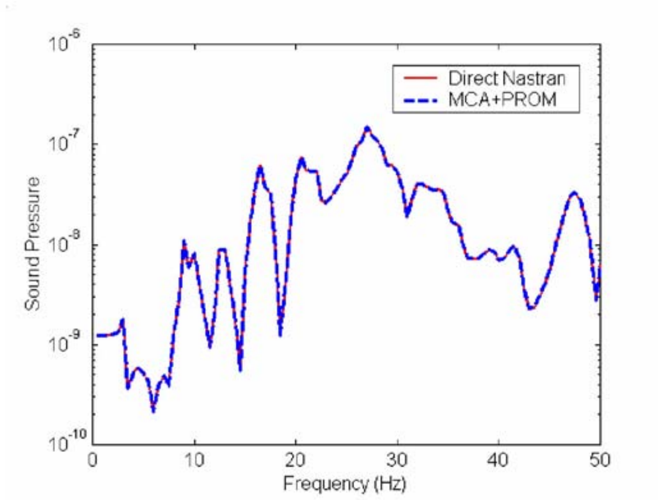
Eigen-Frquencies: MCA+PROM vs. Nastran



- Maximum error is 0.06% in the frequency range of 0-100 Hz.

Accuracy of Forced Response

Sound Pressure Level: MCA+PROM vs. Nastran



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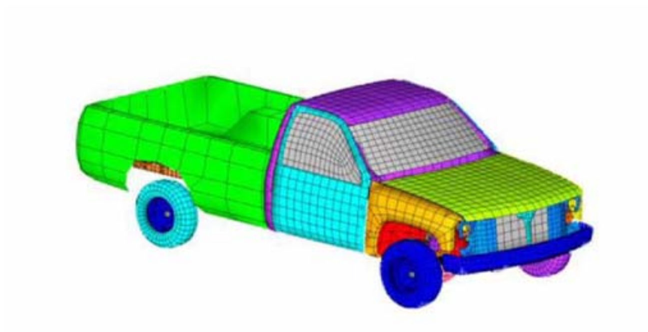
Vehicle Model: Design Optimization



- **Objective: Minimize sound pressure level at driver's ear**
- **5 Design Variables**

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Observations

- Two acoustic modes at 95.9 Hz and 128.3 Hz
- Approximately 1050 modes in the 0 – 300 Hz range
- 500 significant modes were identified and used in modal model

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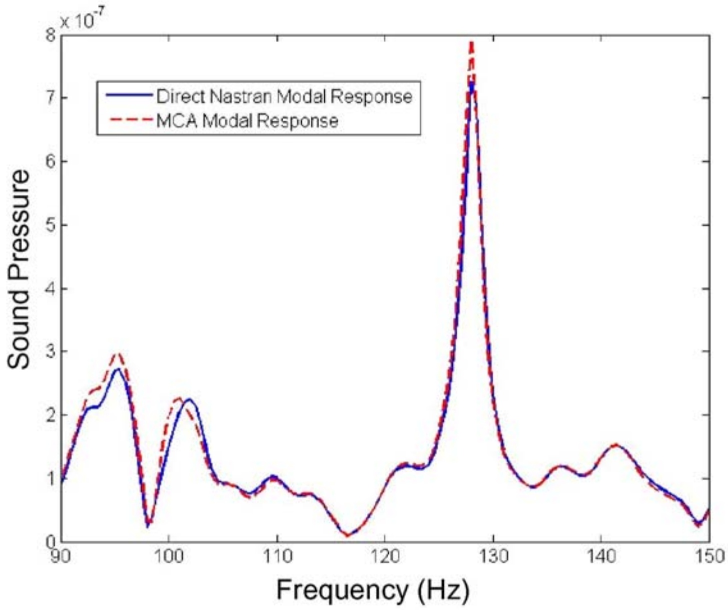
Accuracy of MCA Method



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Optimization Results

Design Variable	Description (thickness of)	Baseline Design (mm)	Optimal Design (mm)	% Change
1	**Chassis**	3.137	3.218	**+2.6**
2	**Chassis cross link**	3.611	5.225	**+44.7**
3	**Cabin**	2.370	3.555	**+50.0**
4	**Bed**	2.500	2.170	**-13.2**
5	**Left and right doors**	1.240	1.730	**+39.5**

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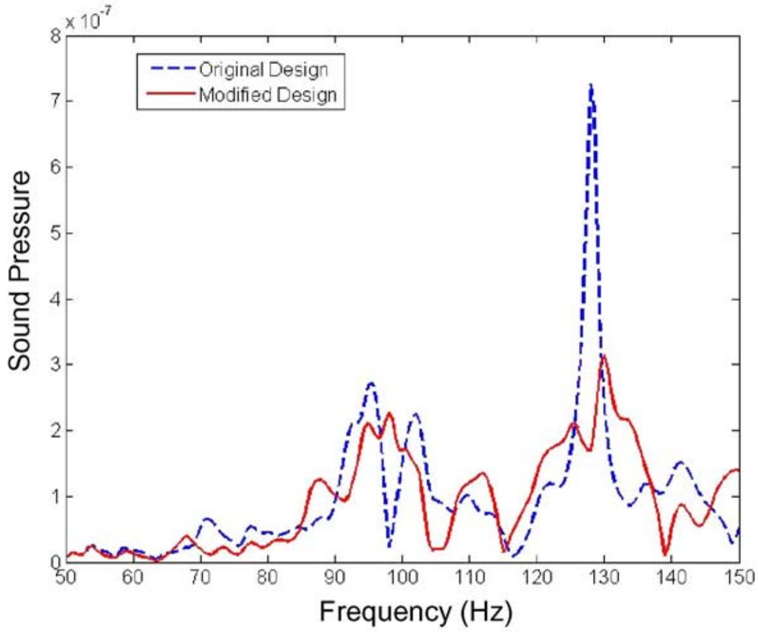
Optimization Results



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Computational Savings in Optimization

- NASTRAN Modal solution: 450 sec
- MCA: 100 sec
- Kriging: 25 sec

Optimization Scheme: Global-Local; 1000 design evaluations (D.E.)

- NASTRAN: $\underbrace{450}_{\text{sec/D.E.}} * \underbrace{1000}_{\text{D.E.}}$ = 450,000 sec or 125 hrs
- PROM+MCA+Kriging: $450 + 100 * \underbrace{5}_{\text{Design Variables}} + 25 * 1000$ = 125,950 sec or 7.2 hrs

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Summary and Conclusions

- An efficient re-analysis method for optimization studies involving the dynamic response of large-scale structures has been presented.
- The method integrates:
  - A modified Combined Approximations (MCA) method,
  - Kriging interpolation and
  - The Parametric Reduced Order Modeling (PROM) approaches.
- A vehicle model was used to demonstrate the accuracy and efficiency of the proposed method.

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