Design of Crankshaft Main Bearings under Uncertainty

Zissimos P. Mourelatos
Mechanical Engineering Department
Oakland University
Rochester, MI 48309, USA

ABSTRACT

A probabilistic analysis is presented for studying the variation effects on the main bearing performance of an I.C. engine system, under structural dynamic conditions. The analysis is based on surrogate models (metamodels), which are developed using the kriging method. The metamodels provide an efficient and accurate substitute to the actual engine bearing simulation models. The bearing performance is based on a comprehensive engine system dynamic analysis which couples the flexible crankshaft and block dynamics with a detailed main bearing elastohydrodynamic analysis. The clearance of all main bearings and the oil viscosity comprise the random design variables. Probabilistic analyses are performed to calculate the mean, standard deviation and probability density function of the bearing performance measures. A Reliability-Based Design Optimization (RBDO) study is also conducted for optimizing the main bearing performance under uncertainty. Results from a V6 engine are presented.

keywords: I.C. engines, crankshaft, main bearing elastohydrodynamic analysis, engine system dynamics, surrogate modeling, design under uncertainty, reliability-based design optimization

1. INTRODUCTION

This paper presents a probabilistic analysis of the main bearing lubrication performance of an operating internal combustion engine. Surrogate models (metamodels) are developed for critical lubrication performance measures based on a detailed dynamic engine simulation solver which couples the structural dynamics of the crankshaft and block with detailed main bearing elastohydrodynamic behavior. The Kriging method [1] is used to generate the metamodels based on a limited number of sample points. Probabilistic analyses are first performed to calculate the main bearing statistical performance in terms of the mean, standard deviation and probability density function of defined bearing performance measures. Subsequently, a probabilistic sensitivity analysis is described for identifying the important random variables. Finally, a Reliability-Based Design Optimization (RBDO) [2,3] study is conducted for optimizing the main bearing performance under uncertainty and results from a V6 engine are presented.

A significant amount of work in the area of elastohydrodynamic (EHD) analysis of connecting rod bearings has been reported in the literature. An integrated system level operating V6 engine simulation model, consisting of flexible crankshaft and engine block dynamics model coupled by an efficient elastohydrodynamic bearing lubrication solver has been presented in [4,5]. A detailed coupling of the crankshaft rigid and flexible body dynamics [6] was used. The work in [4,5] is employed in this paper for calculating the performance measures used in the metamodel generation.

An operating V6 engine represents a complicated non-linear system which is affected by variation in manufacturing processes, operating conditions, material properties, etc. For this study, variability is introduced in some engine design variables and a probabilistic analysis is preformed for the main bearing performance. The clearance at each main bearing and the oil viscosity comprise the random variables. All random variables are assumed normally distributed. The maximum oil film pressure and the percentage of time (the time ratio) within each cycle that a bearing operates with film thickness lower than a user defined threshold value are defined as the performance measures. Eight performance measures are considered. A probabilistic sensitivity analysis is also performed for identifying the most important design variables and for determining the degree to which these design variables influence the performance.

Reliability-Based Design Optimization (RBDO) [2,3] provides an optimum solution in the presence of uncertainty. A mean performance measure is usually optimized subject to the probability of satisfying a constraint being greater than a prescribed reliability level. Deterministic optimal designs that are obtained without considering uncertainty are usually unreliable. In contrast, input variation is fully accounted for in RBDO through probability distributions which describe the stochastic nature of design variables and model parameters [2,3,7,8]. In this paper, an RBDO study of the main bearing lubrication performance is presented based on a newly developed single-loop RBDO algorithm [8].

2. DEVELOPMENT OF A SYSTEM-LEVEL ENGINE BEARING SIMULATION MODEL

The integrated system-level engine bearing simulation model developed in [4-6] is used. This model consists of a flexible crankshaft model and a flexible engine block model connected by a detailed elastohydrodynamic lubrication model. A brief description is given in this section.

The Craig-Bampton method [9] is employed for reducing the physical Degrees of Freedom (DOF) for both the crankshaft and the engine block. The reduced dynamic equations of motion are expressed as

$$\left[\overline{M}^i\right]\begin{Bmatrix}\ddot{\alpha}^i\\ \ddot{x}_r^i\end{Bmatrix}+\left[\overline{C}^i\right]\begin{Bmatrix}\dot{\alpha}^i\\ \dot{x}_r^i\end{Bmatrix}+\left[\overline{K}^i\right]\begin{Bmatrix}\alpha^i\\ x_r^i\end{Bmatrix}=\left\{\overline{F}^i\right\};\ i=b,c \tag{1}$$

where $\overline{M}$, $\overline{C}$ and $\overline{K}$ are the reduced mass, damping and stiffness matrices respectively, and $\overline{F}$ is the modal force vector. The superscripts "*b*" and "*c*" denote the engine block and crankshaft, respectively. Details are given in [4,6].

The flexible crankshaft and flexible block are interacting through a set of distributed nonlinear springs and dampers which are represented by a set of stiffness and damping matrices for each bearing. The stiffness and damping matrices are determined through a detailed lubrication model [5].

The lubricating oil film pressure distribution for each journal bearing is described by the following Reynolds equation,

$$\frac{\partial}{\partial z}\left(\underline{h}^3\frac{\partial \underline{P}}{\partial z}\right)+\frac{\partial}{\partial x}\left(\underline{h}^3\frac{\partial \underline{P}}{\partial x}\right)=6\mu\omega R\frac{\partial \underline{h}}{\partial x}+12\mu\dot{\underline{h}} \tag{2}$$

where $\underline{P} = \underline{P}(x,z,t)$ is the oil film hydrodynamic pressure and $\underline{h} = \underline{h}(x,z,t)$ is the lubricant film thickness. Figure 1 shows the used notation. Eq. (2) is discretized and solved numerically for the hydrodynamic pressure field subject to an imposed known pressure at the two bearing ends and along the oil grooves. The Reynolds cavitation condition is used to account for oil cavitation.

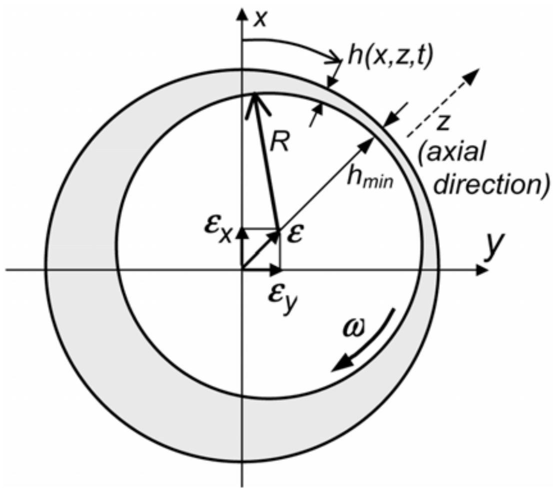

Figure 1. Journal bearing notation

A linear perturbation approach is used to solve Eq. (2). At each time step, the film thickness and squeeze film thickness distributions are assumed equal to $\underline{h} = \underline{h}_0 + \underline{\delta}$ and $\dot{\underline{h}} = \dot{\underline{h}}_0 + \dot{\underline{\delta}}$, respectively, where $\underline{h}_o$ and $\dot{\underline{h}}_o$ are the distributions from the previous time step and $\underline{\delta}$ and $\dot{\underline{\delta}}$ are their corresponding perturbations. In this case, the pressure distribution is approximated as

$$\underline{P} = \underline{P}_o + \frac{\partial \underline{P}}{\partial \underline{h}}\underline{\delta} + \frac{\partial \underline{P}}{\partial \dot{\underline{h}}}\dot{\underline{\delta}}, \tag{3}$$

where the terms $[K] = \frac{\partial \underline{P}}{\partial \underline{h}}$ and $[C] = \frac{\partial \underline{P}}{\partial \dot{\underline{h}}}$ constitute the oil film stiffness and damping matrices, respectively.

Under the assumption of small perturbation values for $\underline{\delta}$ and $\dot{\underline{\delta}}$, Eq. (2) yields the following perturbation equations [4]

$$\frac{\partial}{\partial z}\left(\underline{h}^3 \frac{\partial}{\partial z}\left(\underline{P}_o\right)\right) + \frac{\partial}{\partial x}\left(\underline{h}^3 \frac{\partial}{\partial x}\left(\underline{P}_o\right)\right) = 6\mu\omega R \frac{\partial \underline{h}_o}{\partial x} + 12\mu \dot{\underline{h}}_o \tag{4}$$

$$\frac{\partial}{\partial z}\left(\underline{h}^3 \frac{\partial}{\partial z}\left([K]\underline{\delta}\right)\right) + \frac{\partial}{\partial x}\left(\underline{h}^3 \frac{\partial}{\partial x}\left([K]\underline{\delta}\right)\right) = 6\mu\omega R \frac{\partial \underline{\delta}}{\partial x} - \frac{\partial}{\partial z}\left(3\underline{h}_o{}^2 \underline{\delta} \frac{\partial}{\partial z}\left(\underline{P}_o\right)\right) - \frac{\partial}{\partial x}\left(3\underline{h}_o{}^2 \underline{\delta} \frac{\partial}{\partial x}\left(\underline{P}_o\right)\right) \tag{5}$$

$$\frac{\partial}{\partial z}\left(\underline{h}^3 \frac{\partial}{\partial z}\left([C]\dot{\underline{\delta}}\right)\right) + \frac{\partial}{\partial x}\left(\underline{h}^3 \frac{\partial}{\partial x}\left([C]\dot{\underline{\delta}}\right)\right) = 12\mu\dot{\underline{\delta}} \tag{6}$$

Equations (4), (5) and (6) constitute the governing equations for the hydrodynamic oil film pressure distribution, stiffness matrix, and damping matrix, respectively. They are all solved numerically using a finite difference method. Details can be found in [4,5].

If the oil film domain is discretized with N nodes, the explicit form of the stiffness matrix is

$$[K] = \begin{bmatrix} \frac{\partial p_1}{\partial h_1} & \frac{\partial p_1}{\partial h_2} & \cdots & \frac{\partial p_1}{\partial h_N} \\ \frac{\partial p_2}{\partial h_1} & \frac{\partial p_2}{\partial h_2} & \cdots & \frac{\partial p_2}{\partial h_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial p_N}{\partial h_1} & \frac{\partial p_N}{\partial h_2} & \cdots & \frac{\partial p_N}{\partial h_N} \end{bmatrix}. \tag{7}$$

A similar expression holds for the damping matrix $[C]$. The ij element of the stiffness (damping) matrix represents the pressure change in node i due to a unit oil film thickness (squeeze film velocity) change at node j. The size of the fully populated oil film stiffness and damping matrices is large in order to accurately capture the highly nonlinear oil film pressure distribution, particularly for high journal eccentricity and journal misalignment conditions. For computational efficiency reasons, a diagonalization procedure of the oil film stiffness and damping matrices has been implemented [5]. The oil film stiffness and damping matrices are used in the system level equations between the crankshaft and the block.

For the solution of Eqs (4) through (6), the oil film thickness distribution $h\ x,z,t$ is needed. It is derived as a function of the generalized coordinates of the block x_b and the crankshaft x_c as

$$h(x,z,t) \quad c \quad T_c\ \{x_c\} \quad [T_b]\{x_b\} \tag{8}$$

where T_c and T_b are appropriate transformation matrices [4]. The magnitude of the interaction forces {Q} created between the flexible block and the flexible crankshaft, is a function of the oil film thickness (Eq. (8)) and the stiffness (Eq. (7)) and damping matrices. These interaction forces provide the coupling mechanism between the two components.

In general, the system of reduced dynamic equations for the combined crankshaft and block are written as

$$\overline{M}\ \{\ddot{q}\} \quad [\overline{C}]\{\dot{q}\} \quad [\overline{K}]\{q\} \quad \{\overline{Q}_e\} \quad \{\overline{Q}\} \tag{9}$$

where q are the generalized coordinates. The reduced matrices $\overline{M}$, $\overline{C}$ and $\overline{K}$, and the reduced generalized external forces vector $\overline{Q}_e$ are given in [9]. The expression for the reduced vector $\overline{Q}$ of the lubrication interaction forces, is provided in [4].

The nonlinear system of reduced Eqs (9) is solved in the time domain using a modified Newmark method for time integration, which includes a Newton-Raphson iteration within each time step. Details for the scheme selection can be found in [4]. An operating V6 engine is analyzed by employing the outlined engine dynamic simulation model. Figure 2 shows the used finite element model of the engine block and crankshaft and the combustion pressure.

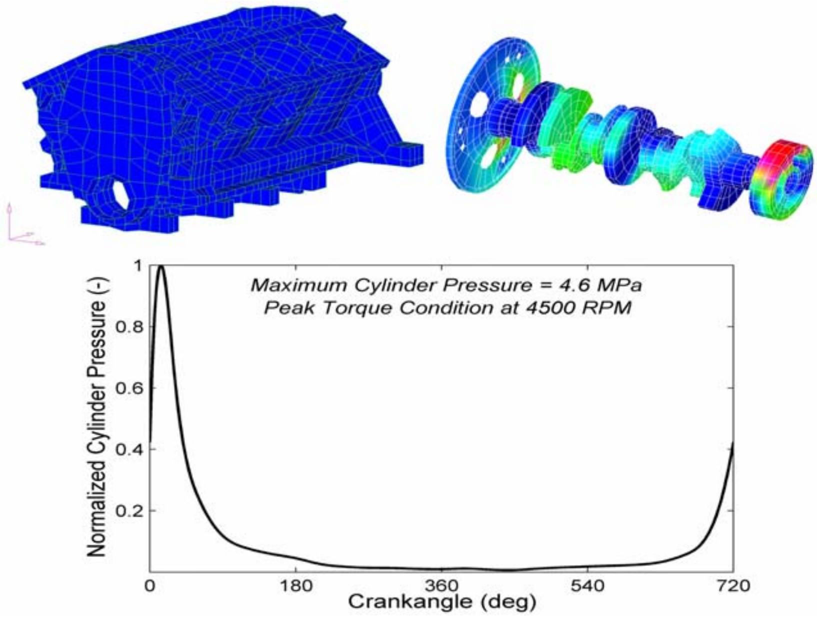

Figure 2. Crankshaft and block FE mesh and cylinder pressure

3. METAMODEL DEVELOPMENT AND VALIDATION

Metamodels have been developed for selected bearing performance measures of an operating V6 engine. The system-level engine dynamic solver of the previous section was employed for computing the performance measures at all used sample points. The operating V6 engine represents a typical system-level automotive application. It is a highly non-linear and complicated system, which involves a large number of degrees of freedom (DOF).

The maximum oil pressure over a cycle is crucial in selecting bearing materials with high enough fatigue resistance for protecting the bearing against failure under an overlay fatigue mode and is a critical indicator of the bearing capacity. For this reason, four of the chosen performance measures ($press_1$, $press_2$, $press_3$, and $press_4$) are the maximum oil pressure over a cycle at each main bearing. The percentage of time (time ratio) within each cycle that a bearing operates with a lower film thickness than a user defined threshold constitutes the other four performance measures $ratio_1$, $ratio_2$, $ratio_3$, and $ratio_4$. This time ratio indicates the severity of the bearing working condition. Eight metamodels have been developed for the defined performance measures. The initial clearance between the crankshaft and each bearing (C_1, C_2, C_3, C_4) and the oil viscosity (VIS) comprise the five random variables which are assumed to be normally distributed and uncorrelated.

The engine dynamic solver is computationally intensive. A single run of the system-level engine simulation requires 7 hours on a SUN workstation or 45 minutes on a high speed SGI supercomputer. The required computational time therefore, makes a probabilistic analysis practically infeasible if the engine system model is used directly. However, the metamodels make the probabilistic analysis feasible since they significantly reduce the computational time to evaluate the performance measures for a given set of random variables. The Kriging method [1] is used to create the metamodels.

Table 1 shows the utilized mean values and standard deviations for all five random variables. Their ranges are representative for industrial applications. An Optimal Symmetric Latin Hypercube (OSLH) algorithm [10] is employed for constructing high quality metamodels with a relatively small number of samples.

Table 1. Definition of random variables for the V6 engine application

	Mean (μ)	**Minimum**	**Maximum**	**Standard Deviation (σ)**	**Coefficient of Variation**
C_1(μm)	30	15	45	5	16.67%
C_2(μm)	30	15	45	5	16.67%
C_3(μm)	30	15	45	5	16.67%
C_4(μm)	30	15	45	5	16.67%
VIS(Pa.s)	0.01	0.0058	0.0142	0.0014	14%

A data set consisting of 200 OSLH sample data points is employed. The engine solver is used for computing the values for the performance variables at all 200 sample points. Collecting data for the 200 sample points takes about 130 CPU hours of simulation on an SGI supercomputer. The accuracy of the developed metamodels has been validated.

Figure 3 shows the PDF of the $press_3$ performance measure, calculated with the Monte Carlo Simulation (MCS) method. The PDF demonstrates a non-normal type of distribution. Because the V6 engine is a highly non-linear system, the random distribution of $press_3$ is not normal, although the input random variables are assumed normal and uncorrelated. Similar non-normal behavior is observed for the other performance measures. The developed metamodels are used to perform probabilistic analyses for the operating V6 engine, efficiently. A sensitivity analysis based on Monte Carlo sampling and a Reliability-Based Design Optimization (RBDO) study, are presented next.

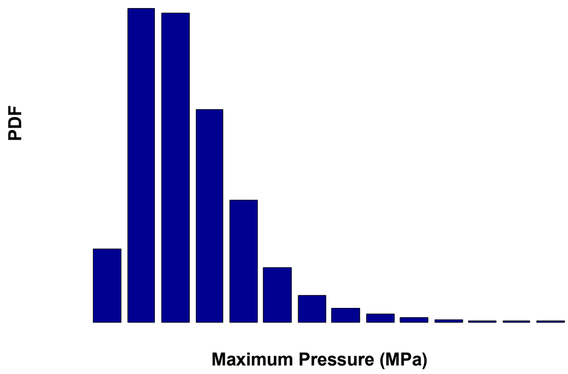

Figure 3. PDF of performance measure $press_3$

4. PROBABILISTIC SENSITIVITY ANALYSIS

Probabilistic sensitivity analysis gives the sensitivity of probability of failure P_f with respect to the mean and standard deviation of the input random variables. In probabilistic analysis, a performance function (or performance measure) is defined as $y(\mathbf{X}) = y(X_1, X_2, \ldots, X_d)$ where $\mathbf{X} = \{X_1, X_2, \ldots, X_d\}^T$ is the vector of *d* random variables. A limit state function $g(\mathbf{X}) = y(\mathbf{X}) - y_0 = 0$ where y_0 is a particular value of y, separates the performance space into a failure region [$g \le 0$] and a safe region [$g>0$]. Given the joint probability density function (PDF), $f_x(\mathbf{X})$, the probability of failure is

$$P_f(y<y_0) = \int \cdots \int_{\Omega} f_x(\mathbf{X})\, d\mathbf{X} \tag{10}$$

where Ω denotes the failure region $g \le 0$. The probability of failure can be interpreted as the probability of violating a performance measure.

Two dimensionless probabilistic sensitivity coefficients have been proposed [11]; the mean sensitivity coefficient, S_{μ_i}, and the standard-deviation sensitivity coefficient, S_{σ_i}. They are defined as

$$S_{\mu_i} = \frac{\partial P_f / P_f}{\partial \mu_i / \sigma_i} \tag{11}$$

$$S_{\sigma_i} = \frac{\partial P_f / P_f}{\partial \sigma_i / \sigma_i} \tag{12}$$

where μ_i and σ_i are the mean and standard deviation of random variable X_i. S_{μ_i} and S_{σ_i} are the nondimensional sensitivity measures of P_f with respect to the mean and standard deviation respectively, for random variable X_i. The probability sensitivity coefficients can be positive, negative, or of zero value. A large magnitude sensitivity coefficient indicates that the corresponding random variable is important.

The sensitivity coefficients of Eqs (11) and (12) are employed in the dynamic analysis of the operating V6 engine to determine the sensitivity of the performance measures with respect to the mean and standard deviation of each input random variable. The five random variables C_1, C_2, C_3, C_4 and VIS are assumed normally distributed and uncorrelated. A large number of samples is generated and the corresponding performance measures are evaluated using the metamodels. The random variables C_3 and VIS have been identified as the most important ones.

5. RELIABILITY-BASED DESIGN OPTIMIZATION OF MAIN BEARING PERFORMANCE FOR A V6 ENGINE

In deterministic design we assume that there is no uncertainty in the design variables and/or modeling parameters. Therefore, there is no variability in the simulation outputs. However, there exists inherent input and parameter variation that results in output variation. Deterministic optimization typically yields optimal designs that are pushed to the limits of design constraint boundaries, leaving little or no room for tolerances (uncertainty) in manufacturing imperfections, modeling and design variables. Therefore, deterministic optimal designs that are obtained without taking into account uncertainty are usually unreliable. Input variation is fully accounted for in Reliability-Based Design Optimization (RBDO) [2,3]. Probability distributions describe the stochastic nature of the design variables and model parameters. Variations are represented by standard deviations (typically assumed to be constant) and a mean performance measure is optimized subject to probabilistic constraints.

5.1. OVERVIEW OF RELIABILITY-BASED DESIGN OPTIMIZATION (RBDO) METHODS

A deterministic optimization problem is converted to a probabilistic optimization or RBDO, problem if the inequality constraints of the former are satisfied probabilistically. In such a case, the probability of satisfying an inequality constraint must be greater than a prescribed reliability level which is usually very high. A typical RBDO problem is then formulated as

$$\text{s.t.} \quad P\ G_i\ \mathbf{d},\mathbf{X},\mathbf{P}\quad 0 \quad R \quad i\ \ 1,2,\ldots,n\,, \quad i\ \ 1,2,\ldots,n \qquad (13)$$
$$\mathbf{d}^L\ \ \mathbf{d}\ \ \mathbf{d}^U$$

where $\mathbf{d}$ R^k is the vector of deterministic design variables, $\mathbf{X}$ R^m is the vector of random design variables, $\mathbf{P}$ R^q is the vector of random and deterministic design parameters, f is the objective function and *n, k, m* and *q* are the number of constraints, deterministic design variables, random design variables and design parameters, respectively. According to the used notation, a bold letter indicates a vector, an upper case letter indicates a random variable or random parameter and a lower case letter indicates a realization of a random variable or random parameter. The actual reliability level for the i^{th} deterministic constraint is

$$R_i \quad 1 \quad p_{f_i} \qquad i\ \ 1,2,\ldots,n\,, \qquad (14)$$

where (15)

is the target probability of violating the i^{th} deterministic constraint which is usually very small. Note that in the RBDO formulation of Eq. (13) the design variables include only the means of the random variables. The target probability of failure p_f is usually approximated by the following first-order relation

$$p_f \approx \Phi - \ _t \qquad (16)$$

where $_t$ is the target reliability index and is the standard normal cumulative distribution function.

Problem (13) can be solved using two nested optimization loops (double-loop RBDO method); the design optimization loop (outer) and the reliability assessment loop (inner). The latter is needed for the evaluation of each probabilistic constraint in Eq. (13). For this reason, the double-loop RBDO method is computationally very expensive and therefore, almost impractical for large-scale design problems. There are two different methods for the reliability assessment; the Reliability Index Approach (RIA) [2] and the Performance Measure

Approach (PMA) [3]. Although either approach can be used, PMA is in general more efficient, especially for high reliability problems [12]. Every time the design optimization loop calls for a constraint evaluation, a reliability assessment loop is executed which searches for the MPP in the standard normal space.

If the PMA-based approach is used, the general RBDO problem of Eq. (13) is stated as [12]

$$\text{s.t. } G_{p_i} \quad 0, \; i \quad 1,2,...,n \qquad (17)$$
$$\mathbf{d}^L \quad \mathbf{d} \quad \mathbf{d}^U$$

where each probabilistic constraint is transformed to an equivalent inequality constraint involving the performance measure G_p which is calculated from the following reliability minimization problem

$$(18)$$
$$\text{s.t } \|\mathbf{U}\| = \quad {}_t$$

where the vector ***U*** represents the random variables in the standard normal space.

As shown from Eq. (17), the PMA-based RBDO formulation involves nested optimization loops, which may hinder on its computational efficiency and convergence properties. The same holds for the RIA-based RBDO formulation. To improve the computational efficiency, two new classes of RBDO formulations have been recently proposed. The first class decouples the RBDO process into a sequence of a deterministic design optimization followed by a set of reliability assessment loops [7]. The series of deterministic and reliability loops is repeated until convergence. The second class of RBDO methods converts the problem into an equivalent, single-loop deterministic optimization [8] providing therefore, a substantial computational advantage. The single-loop RBDO method of [8] is used in the case study of the next section.

5.2. APPLICATION

An RBDO study of the main bearing lubrication performance of an operating V6 engine is presented here. The maximum oil film pressure for each bearing is minimized subject to each maximum pressure being below a specified value and the oil film time ratio for each bearing being less than a specified value. The oil film time ratio is defined as the percentage of time within each cycle that a bearing operates with a film thickness less than a threshold. Two random variables are considered; the radial clearance C of each bearing and the oil viscosity VIS. The objective function is simply the sum of the means of maximum oil pressure in each bearing. The RBDO problem is stated as

$$\min_{\mu_C, \mu_{VIS}} f = \quad {}_{press_1} + \quad {}_{press_2} + \quad {}_{press_3} + \quad {}_{press_4}$$
$$\text{s.t. } P(G_j(\mathbf{X}) \quad 0) \quad R_j \qquad j \quad 1,2...,8$$

$$G_i(\mathbf{X}) = 1 - \frac{press_i}{134} \geq 0 \qquad i = 1,...,4$$
$$G_{4+i}(\mathbf{X}) = 1 - \frac{ratio_i}{0.27} \geq 0 \qquad i = 1,...,4$$
$$; \; 0.0058 Pa*s \quad \mu_{VIS} \quad 0.0142 Pa*s$$
$${}_C \quad 5 \; m \; ,$$
$${}_j \quad 1.28 \text{ or } {}_j \quad 1.52 \qquad for \; j \quad 1,2,...,8$$

where μ_C, μ_{VIS} and σ_C, σ_{VIS} are the mean values and standard deviations respectively, of the two design random variables C and VIS. The bounds for μ_C, μ_{VIS} and σ_C, σ_{VIS} are given in Table 1. For demonstration purposes, the same target reliability index $\beta = 1.28$ or $\beta = 1.52$ is used for all eight constraints. In general, a different target reliability index may be used for each constraint. A reliability index of 1.28 or 1.52 corresponds to a reliability level R of 89.97% or 93.57%, respectively. The first four inequality constraints indicate that the maximum oil pressure in all four bearings must be less than 134 MPa. The last four inequality constraints impose the requirement that the time ratio for each bearing must be less than 0.27.

Figure 4 shows the progress of the single-loop RBDO process in the X space for $\beta = 1.28$. Only the constraints corresponding to $press_2$, $press_3$, $ratio_2$ and $ratio_3$ are plotted. The other constraints are outside the design space of Figure 4. The algorithm starts with an

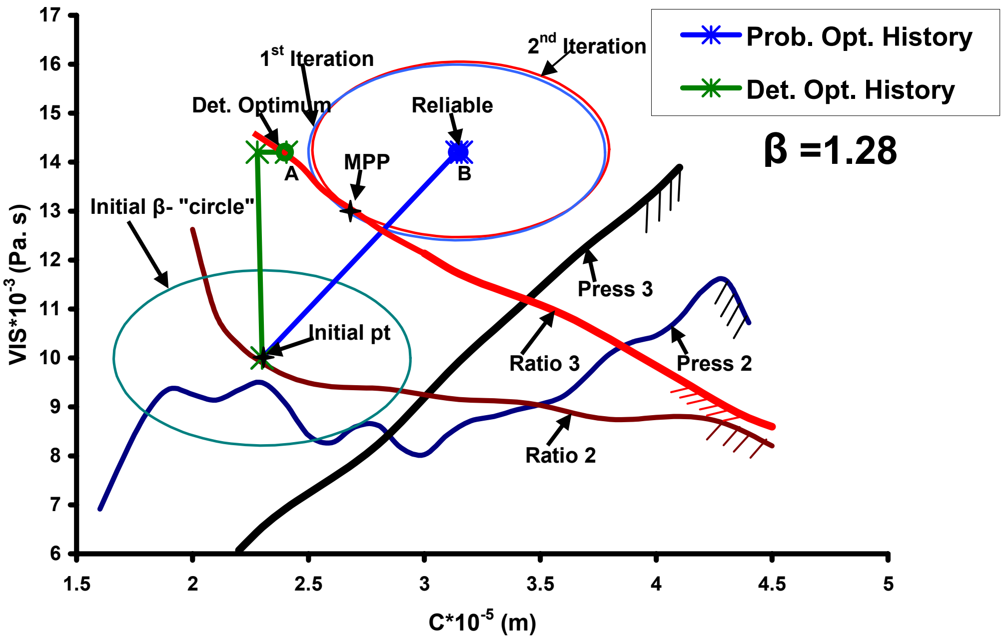

Figure 4. RBDO progress for $\beta = 1.28$

initial point which is infeasible. The corresponding initial β–"circle" is also shown. It should be noted that the β–"circle" does not have a circular shape because the two random variables have different standard deviations. Figure 4 shows the position of the β–"circle" for the two iterations the single-loop algorithm needed [illegible] G_7 [illegible] (unconstrained optimum). In the latter case, the β–"circle" is within the feasible domain without touching its boundaries.

For the $\beta = 1.28$ case, there is an 89.97% chance that a design realization will fall within the β–"circle". We have therefore, an 89.97% reliable design; *all* constraints will be satisfied 89.97% of the time. Figure 5 shows the RBDO results for $\beta = 1.52$. In this case, both constraint G_7 corresponding to ratio$_3$ and constraint G_3 corresponding to press$_3$, are active. The –“circle” is tangent to both G_7 and G_3. With the given variability of C and VIS, indicated by their standard deviations, there is no feasible solution for greater than 1.52. The maximum reliability we can achieve for this problem is therefore, 93.57%.

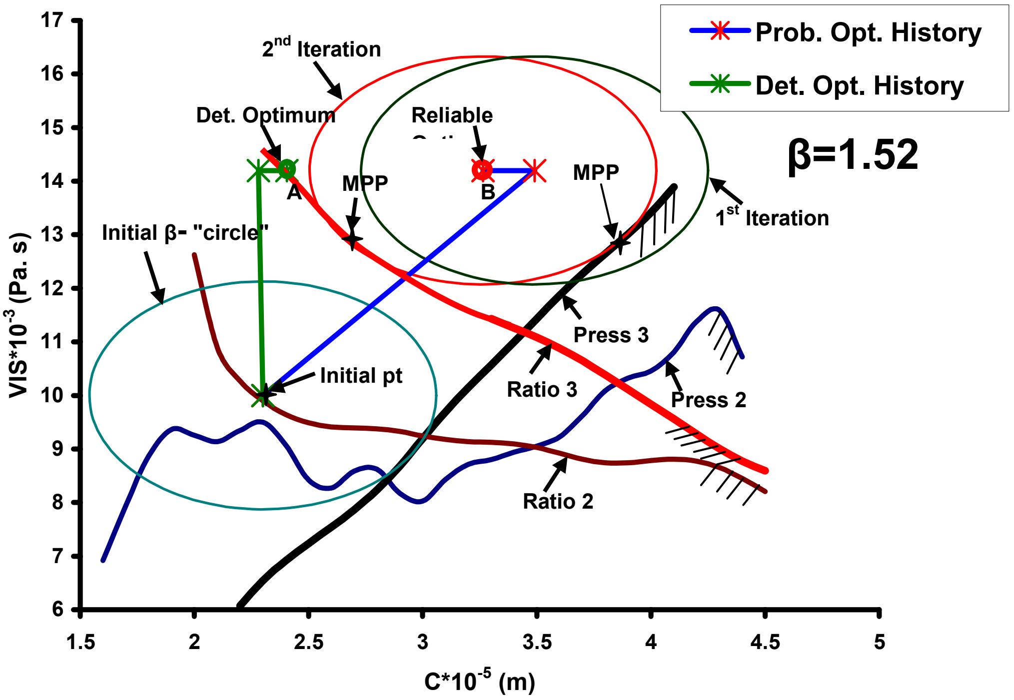

Figure 5. RBDO progress for β 1.52

For comparison purposes, Figures 4 and 5 also show the progress of a deterministic optimization problem with the same initial point. The deterministic optimization converged to the constrained deterministic optimum (point A) in two iterations. Point A has low reliability (around 50%) because uncertainty in the design variables C and VIS will result in design realizations which will violate the ratio$_3$ constraint.

It should be noted that for both the deterministic and probabilistic optima, the mean oil viscosity value was pushed to its upper limit of 0.0142 Pa*s (see Table 2 and RBDO problem statement). This upper limit was responsible for achieving a maximum reliability of only 93.57%. A higher reliability level can be achieved if the viscosity upper limit is raised.

Table 2 illustrates the efficiency of the single-loop RBDO method by comparing it against deterministic optimization. Both need two iterations to converge. The final value of the objective function is 352.09, 366.07 and 370.56 for the deterministic and single-loop methods with β 1.28 and β 1.52, respectively. Both the deterministic and the single-loop RBDO methods required 88 function evaluations. A function evaluation represents a calculation of the objective function or any of the eight constraints. The efficiency of the single-loop RBDO method is the same with that of deterministic optimization. In general, the efficiency of the single-loop method is comparable to the deterministic optimization. It should be noted that the efficiency of any RBDO method can not theoretically exceed the efficiency of the deterministic optimization. For comparison purposes, Table 2 shows the initial point used in both the deterministic and probabilistic optimizations.

Table 2. Summary of results for deterministic and single-loop RBDO methods

	Initial Point	Det. Opt.	Single Loop	
Design Variables			**β=1.28**	**β=1.52**
C_1 (μm)	23	24.055	31.57	32.64
VIS	0.01	0.0142	0.0142	0.0142
Objective				
f(X)	397.6581	352.0961	366.0703	370.5696
Constraints				
$G_1 = 1 - press_1 / 134 \geq 0$	0.2391	0.3211	0.2369	0.2217
$G_2 = 1 - press_2 / 134 \geq 0$	0.0293	0.1782	0.2152	0.0982
$G_3 = 1 - press_3 / 134 \geq 0$	0.1200	0.1492	0.0441	0.0015
$G_4 = 1 - press_4 / 134 \geq 0$	0.6440	0.7238	0.5994	0.5733
$G_5 = 1 - ratio_1 / 0.27 \geq 0$	0.8725	0.9127	0.8894	0.8858
$G_6 = 1 - ratio_2 / 0.27 \geq 0$	0.0146	0.2405	0.2620	0.2633
$G_7 = 1 - ratio_3 / 0.27 \geq 0$	-0.3571	-0.0101	-0.0001	-0.0529
$G_8 = 1 - ratio_4 / 0.27 \geq 0$	0.6734	0.9907	0.8284	0.7302
No. of Iterations		2	2	2
No. of F. E.		88	88	88

REFERENCES

[1] Sacks, J., Welch, W. J., Mitchell, T. J., and Wynn, H. P., “Design and Analysis of Computer Experiments,” *Statistical Science*, 4(4), 409-435, 1989.

[2] Lee, J. O., Yang, Y. O. and Ruy, W. S., “A Comparative Study on Reliability Index and Target Performance Based Probabilistic Structural Design Optimization,” *Computers and Structures*, 80, 257-269, 2002.

[3] Tu, J., Choi, K. K. and Park, Y. H., “A New Study on Reliability-Based Design Optimization”, *ASME Journal of Mechanical Design*, 121, 557-564, 1999.

[4] Ebrat, O., Mourelatos, Z. P., Hu, K., Vlahopoulos, N. and Vaidyanathan, K., “An Elastohydrodynamic Coupling of a Rotating Crankshaft and a Flexible Engine Block,” *Journal of Tribology*, 126, 1-9, 2004.

[5] Ebrat, O., Mourelatos, Z. P., Vlahopoulos, N. and Vaidyanathan, K., “Calculation of Journal Bearing Dynamic Characteristics Including Journal Misalignment and Bearing Structural Deformation,” *Tribology Transactions*, 47, 1-9, 2004.

[6] Hu, K., Mourelatos, Z. P. and Vlahopoulos, N., “A Finite Element Formulation for Coupling Rigid and Flexible Body Dynamics of Rotating Beams,” *Journal of Sound and Vibration*, 253(3), 603-630, 2002.

[7] Du, X. and Chen, W., “Sequential Optimization and Reliability Assessment Method for Efficient Probabilistic Design,” *Proceedings of ASME Design Engineering Technical Conferences (DETC)*, Paper # DETC2002/DAC-34127, 2002.

[8] Liang, J., Mourelatos, Z. P. and Tu, J., “A Single-Loop Method for Reliability-Based Design Optimization,” *Proceedings of ASME Design Engineering Technical Conferences (DETC)*, 2004.

[9] Craig, R. R. and Bampton, M. C. C., “Coupling of Substructures for Dynamic Analysis,” *AIAA Journal*, 6(7), 1313-1319, 1968.

[10] Ye, K. Q., Li, W. and Sudjianto, A., “Algorithm Construction of Optimal Symmetric Latin Hypercube Designs,” *Journal of Statistical Planning and Inference*, 90, 145-159, 2000.

[11] Mohanty, S. and Wu, Y-T., “CDF Sensitivity Analysis Technique for Ranking Influential Parameters in the Performance Assessment of the Proposed High-Level Waste Repository at Yucca Mountain, Nevada, USA,” *Reliability Engineering and System Safety*, 167-176, 2001.

[12] Tu, J., Choi, K. K. and Park, Y. H., “Design Potential Method for Robust System Parameter Design”, *AIAA Journal*, 39(4), 667-677, 2001.