Recent Advances in Re-Analysis Methods for NVH Including Shape and Topology Optimization

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Definitions / Motivation

Excitation ⇨

⇨ Vibratory Response

Vibration Analysis is Computationally Intensive

Deterministic Design Optimization Requires MANY Vibration Analyses

Probabilistic Design Optimization Requires EVEN MORE Vibration Analyses

Observations

In Noise, Vibration and Harshness (NVH) :

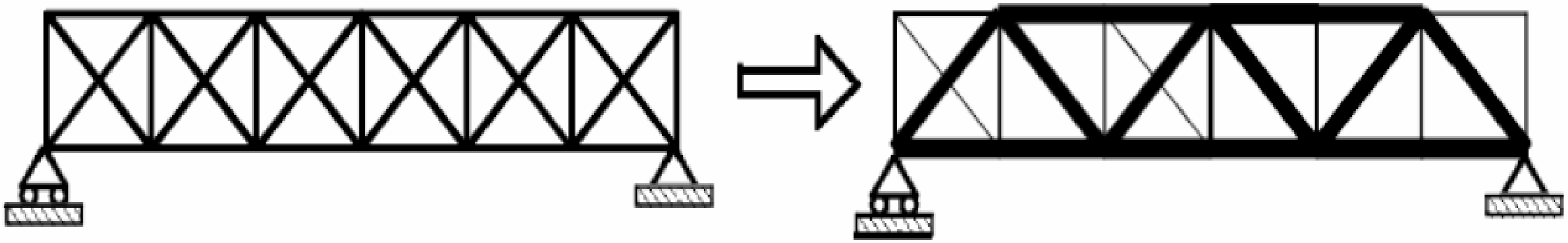
- **Physical partitioning (substructuring) is commonly used**
- **FRF substructuring for interfaces with few DOFs**
- **Craig-Bampton (CMS) substructuring for interfaces with many DOFs**

In design optimization :

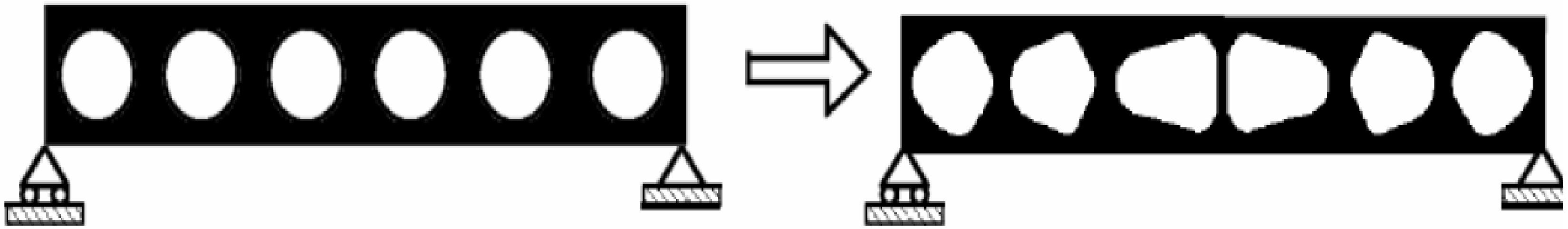
- **Changes can be global or local. The latter are common**
- **“Gauge” (e.g. thickness), shape, or topology changes**

Classification of Optimization Methods

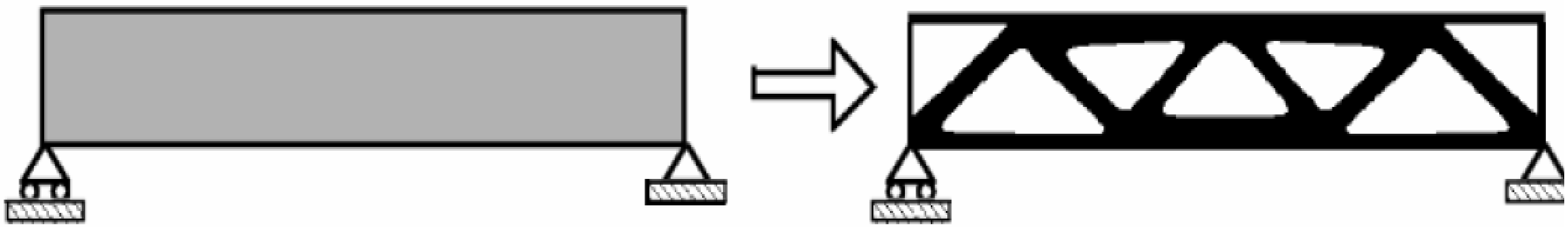
Gauge



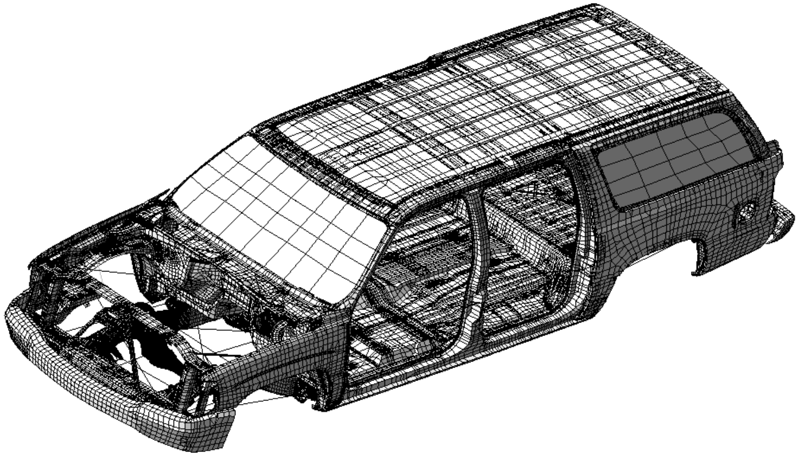
Shape



Topology



Reduced-Order Modeling (Modal Model)
"All-in-One" Approach



Re-analysis Methods

- CDH/VBA Method
- MCA Method
- PROM / MCA Method

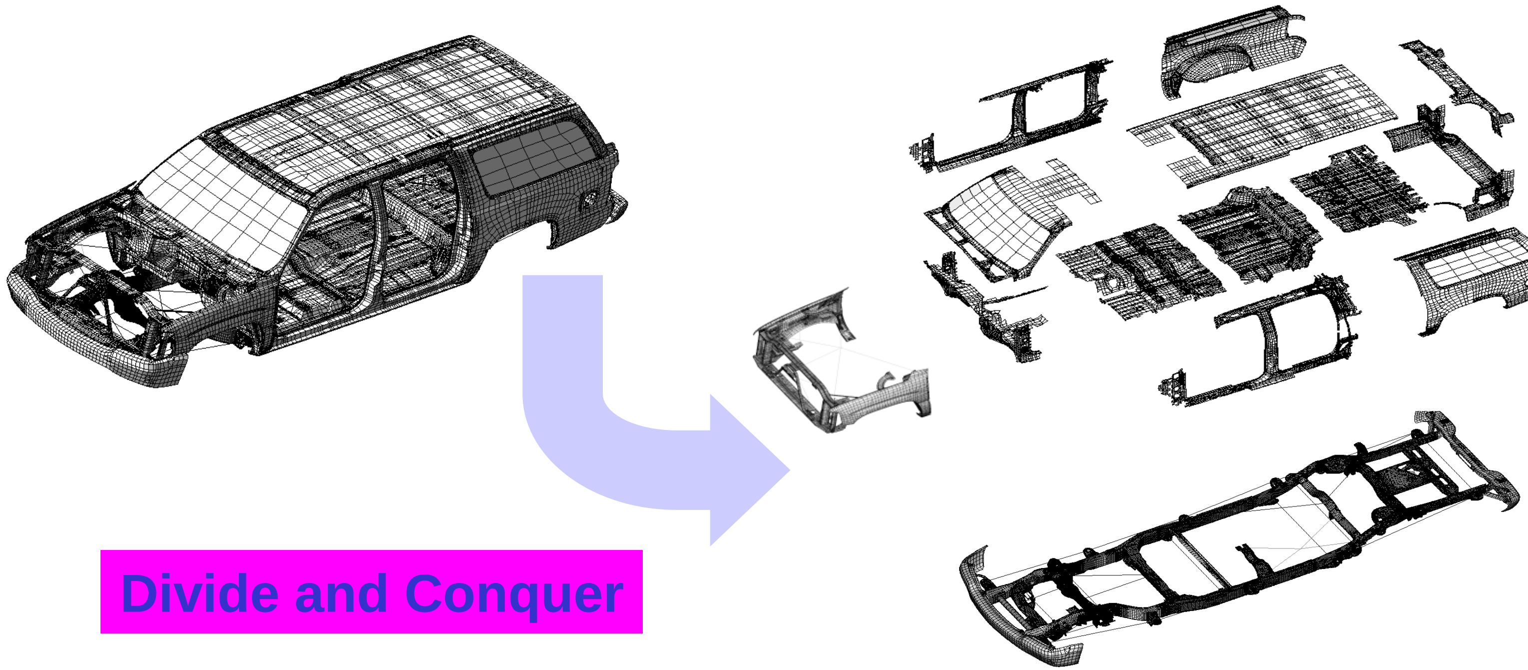
Modal Model: $\left[\boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi} - \omega^2\boldsymbol{\Phi}^T\mathbf{M}\boldsymbol{\Phi}\right]\mathbf{U} = \boldsymbol{\Phi}^T\mathbf{F}$

Practical Issues:

- ➢ Basis $\boldsymbol{\Phi}$ must be recalculated for each new design
- ➢ Calculation of "triple" product $\boldsymbol{\Phi}^T\mathbf{K}\boldsymbol{\Phi}$ can be expensive

Reduced-Order Modeling (Modal Model)

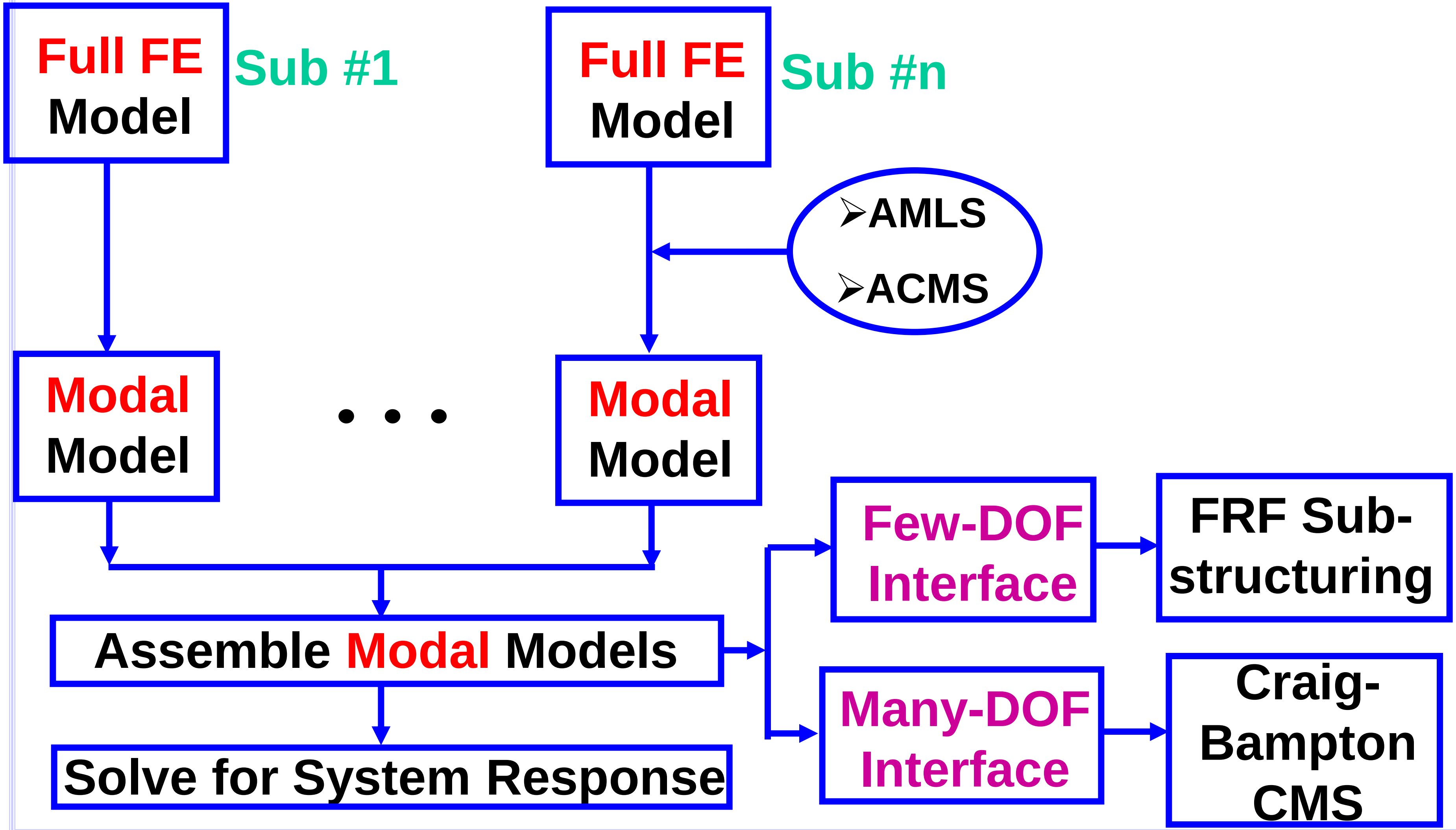
Substructuring Approach



Practical Issues:

- **Basis Φ must be recalculated for each new design**
- **Calculation of “triple” product $\Phi^T K \Phi$ can be expensive**
- **Interface matrices can be large in size**

Substructuring Approach



Overview / Demonstration of Re-Analysis Methods

- Basic Concept of Re-Analysis Methods
- Re-Analysis Methods for Shape Changes
  - Different Examples
  - Shape/Gauge Optimization (Car Example)
- Re-Analysis in Topology Optimization
  - Automotive Door Example

Basic Concept of Re-Analysis Methods

Re-Analysis methods provide approximate eigenvalues and eigenvectors

Conventional Approach

Eigen-analysis of full matrices

$$eig(\mathbf{K}, \mathbf{M}) \Rightarrow \lambda, \mathbf{\Phi}$$

Re-analysis Approach

1. Form reduced basis: $\mathbf{P}$

2. Project system matrices to reduced basis:

$$\mathbf{K}_R = \mathbf{P}^T \mathbf{K} \, \mathbf{P} \qquad \mathbf{M}_R = \mathbf{P}^T \mathbf{M} \, \mathbf{P}$$

3. Eigen-analysis of reduced matrices:

$$eig(\mathbf{K}_R, \mathbf{M}_R) \Rightarrow \tilde{\lambda}^p, \mathbf{\Theta}$$

4. Obtain approximate eigenvectors:

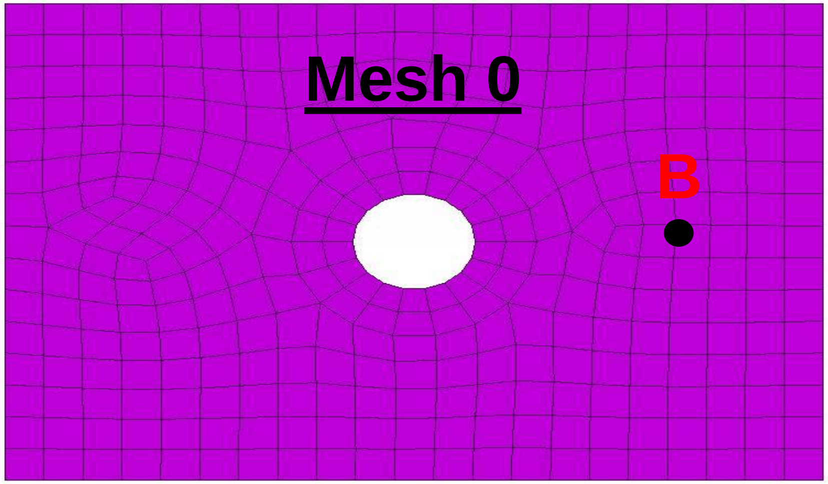
$$\tilde{\mathbf{\Phi}} = \mathbf{P} \, \mathbf{\Theta}$$

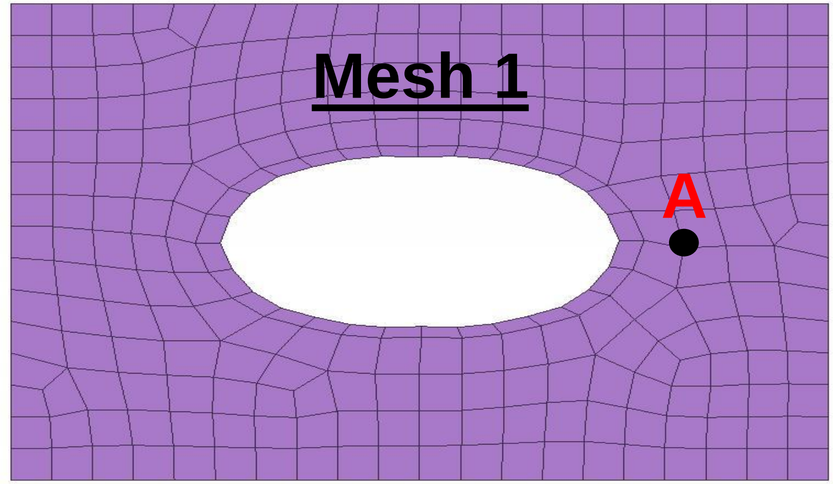
Re-Analysis methods (e.g. CDH, PROM, MCA) are differentiated by definition of reduced basis P

Re-Analysis Methods for Shape Changes

Major Challenge: Different mesh between baseline and modified designs

MCA for Shape Changes: Different DOFs



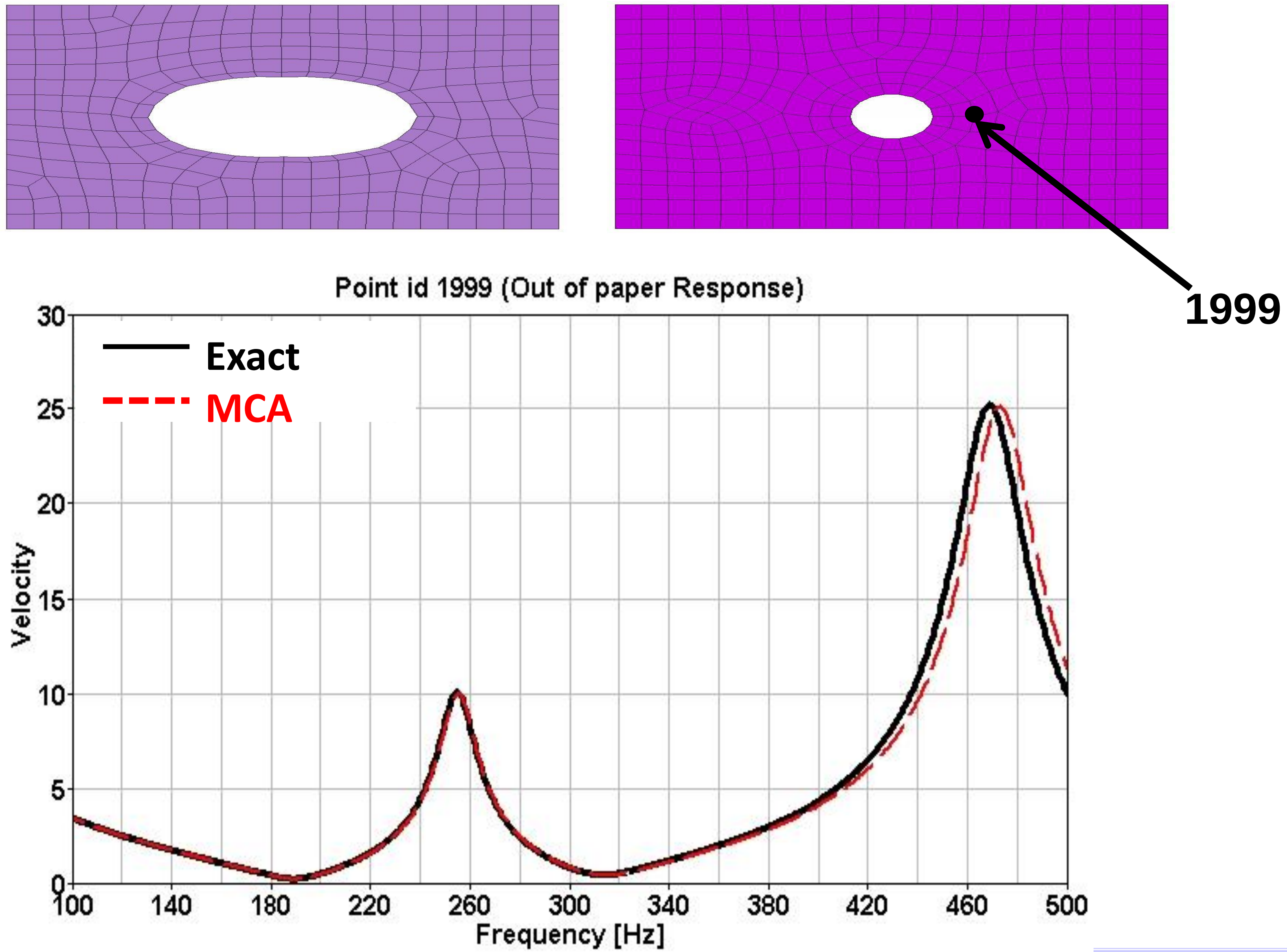


$\Phi_0 \, (n \times m)$ $\Phi_1 \, (l \times m)$

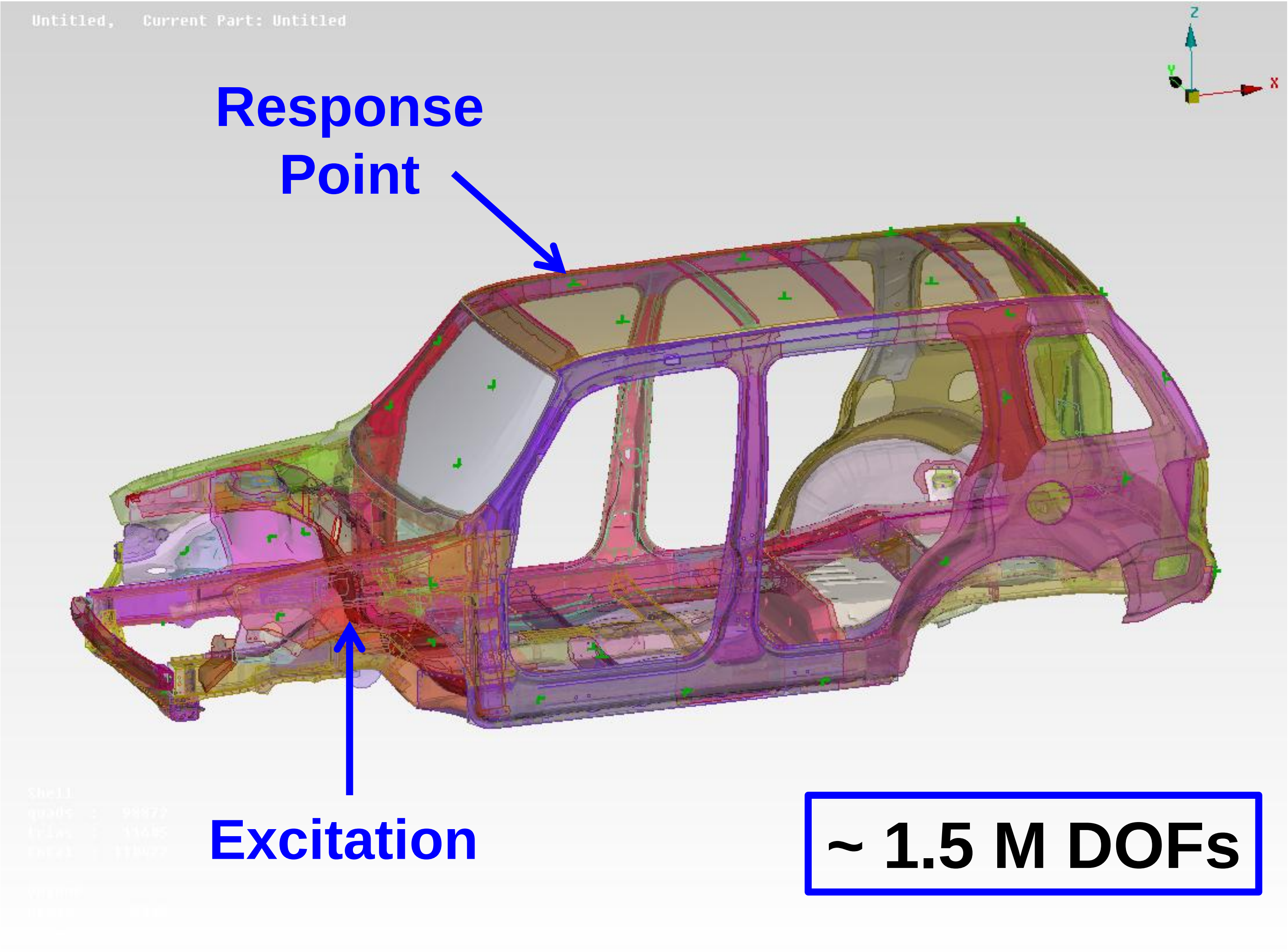
For each node A of FE mesh 1
identify closest node B of ME mesh 0 $\Rightarrow$ $\hat{\Phi}_1 \, (l \times m)$

Use MCA starting with $\hat{\Phi}_1$ to estimate Φ_1

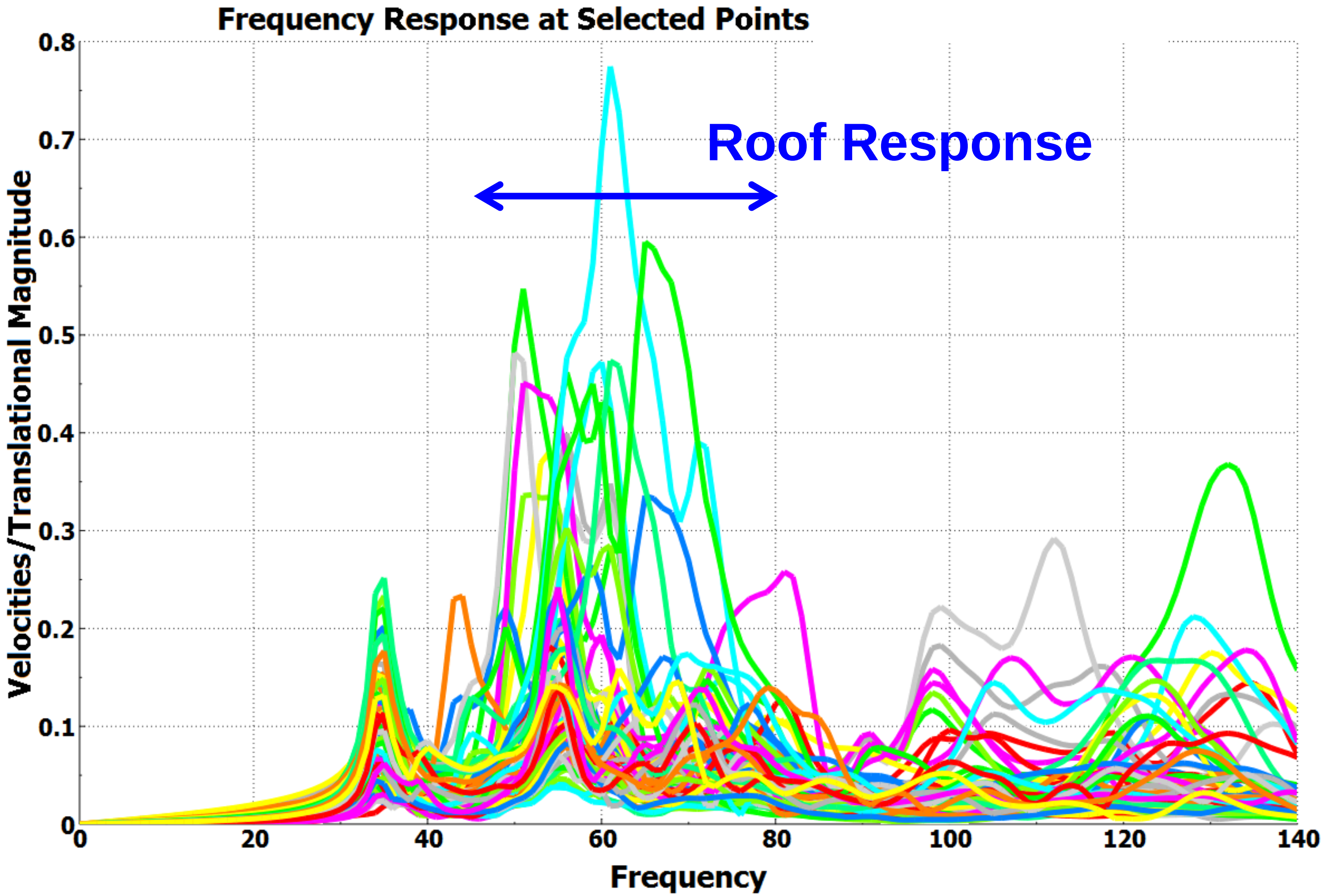
MCA for Shape Changes: Different DOFs



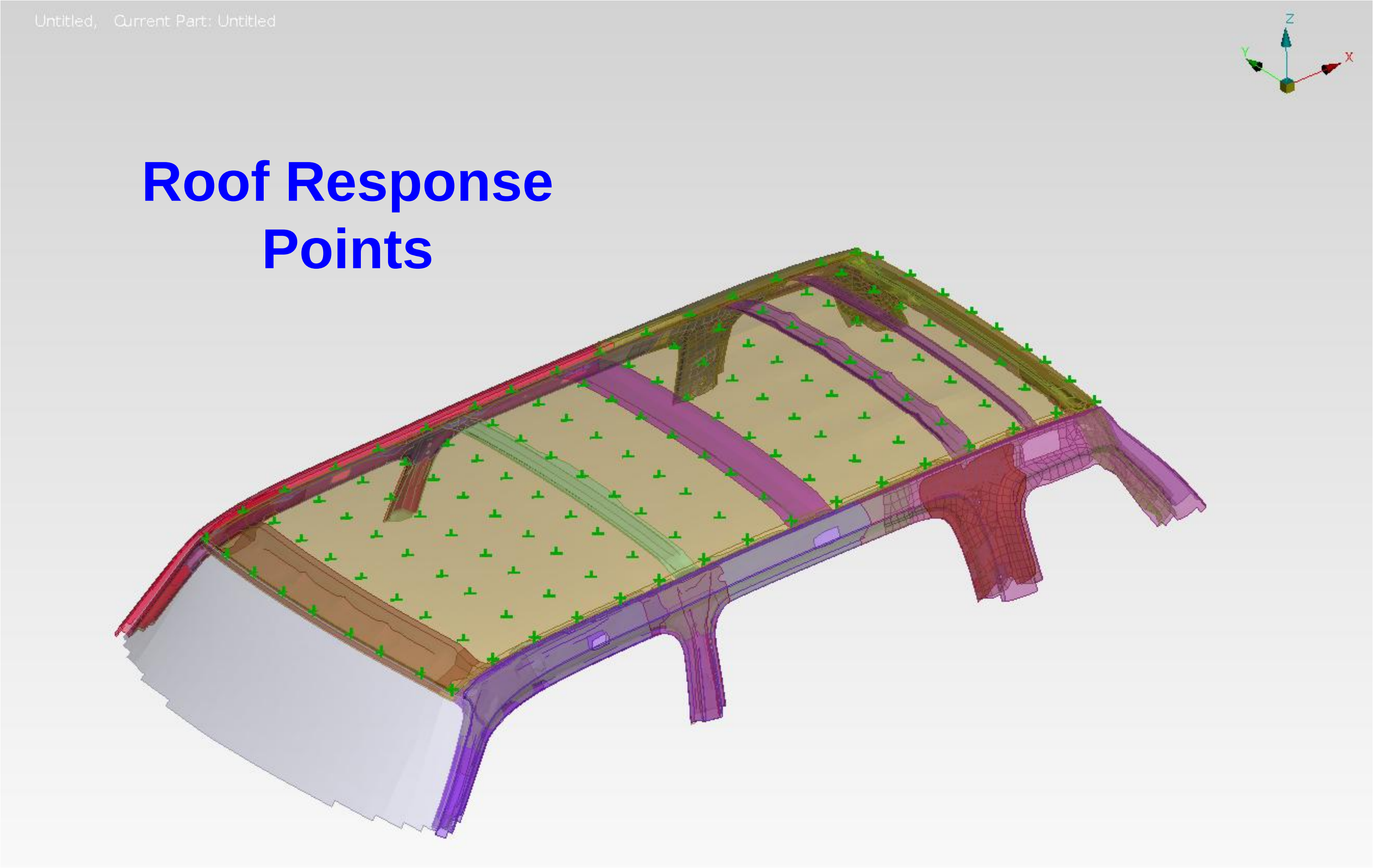
Example of Shape / Gauge Optimization using MCA Re-Analysis



Example of Shape / Gauge Optimization using MCA Re-Analysis

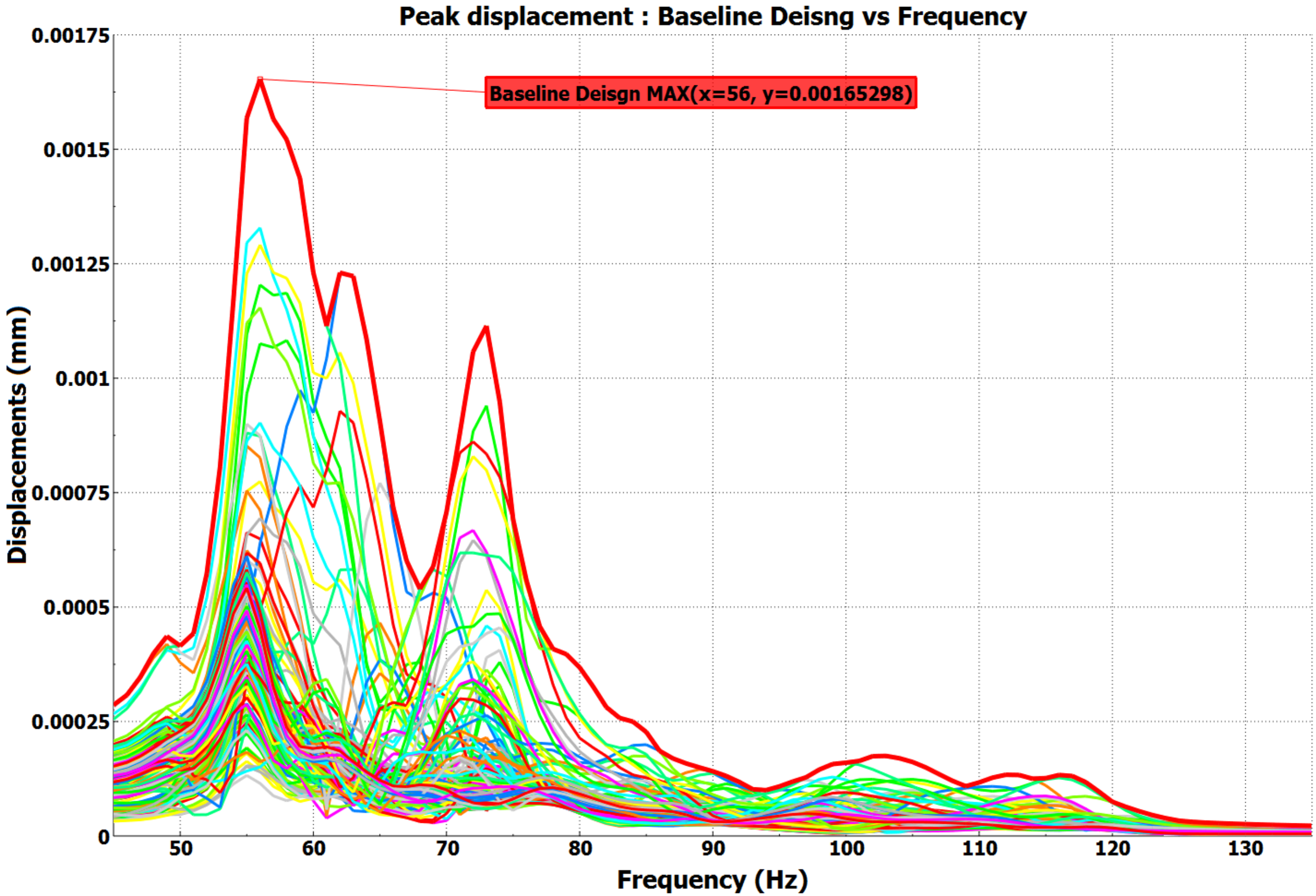


Example of Shape / Gauge Optimization using MCA Re-Analysis



Example of Shape / Gauge Optimization using MCA Re-Analysis

Roof Peak Response Envelope



Design Variables

Morphing Variables

No.	Parameter	Baseline	Lower Bound (mm)	Upper Bound (mm)
1	BPillar_Width	0	-10	40
2	RR_Header_X_Position	0	-150	150
3	Windshield_Header_Position	0	-130	130
4	Roof_Height	0	-130	150
5	7850276_Cross Bow 1	0.75	0.4	2
6	7850263_Cross Bow Main	0.75	0.4	2
7	7824312_ B-Pillar Inner RT Gauge	1.2	0.4	2
8	78502076_Roof Cross Bow 2	0.75	0.4	2
9	7824557_B-Pillar Inner RT Gauge	1.6	0.4	2
10	47071_Floor Cross Member Rea	1.4	0.4	2
11	7810628_Floor Seat Cross Member	0.75	0.4	2
12	92_Roof Cross Bow 3	0.75	0.4	2
13	7850202_Roof Gauge	0.79	0.4	2

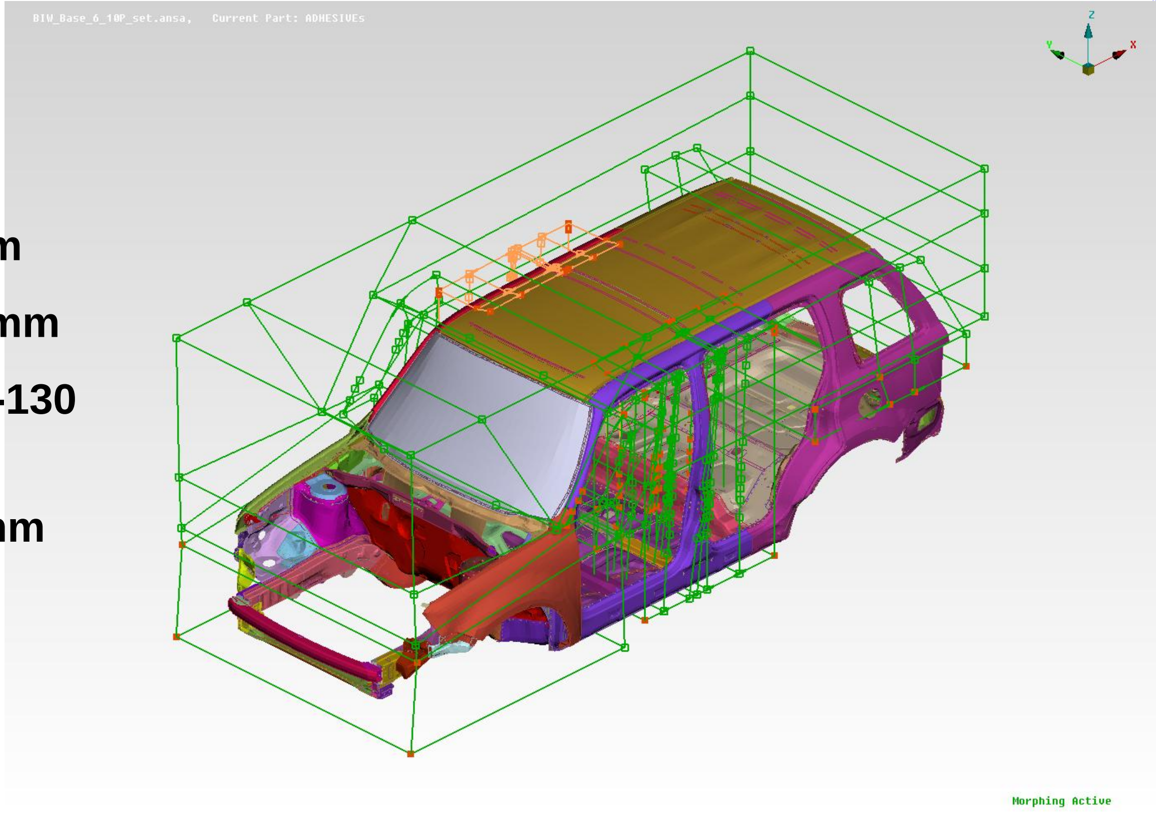
Shape Design Variables

B-Pillar Width = -10 to 40 mm

RR Header Pos = -150 to 150 mm

Windshield Header Position = -130 to +130 mm

Roof Height = -130 to + 150 mm



All values w.r.t. baseline at zero

Optimization Statement

of Design Variables

Find $\mathbf{X} = [X_1 \quad X_2 \quad \cdots \quad X_{13}]$

# of Roof Response Pts

to

$$\min_{\mathbf{X}} \left[\max_{i=1}^{145} \left(Resp_i \right) \right]$$

NSGA II Optimizer

s. t. $Mass \leq Mass_{Nominal}$

where : $Resp = |y(f)|\,, \quad f \in \{45, \quad 140\} \; Hz$

Roof Displacement Magnitude

Approach

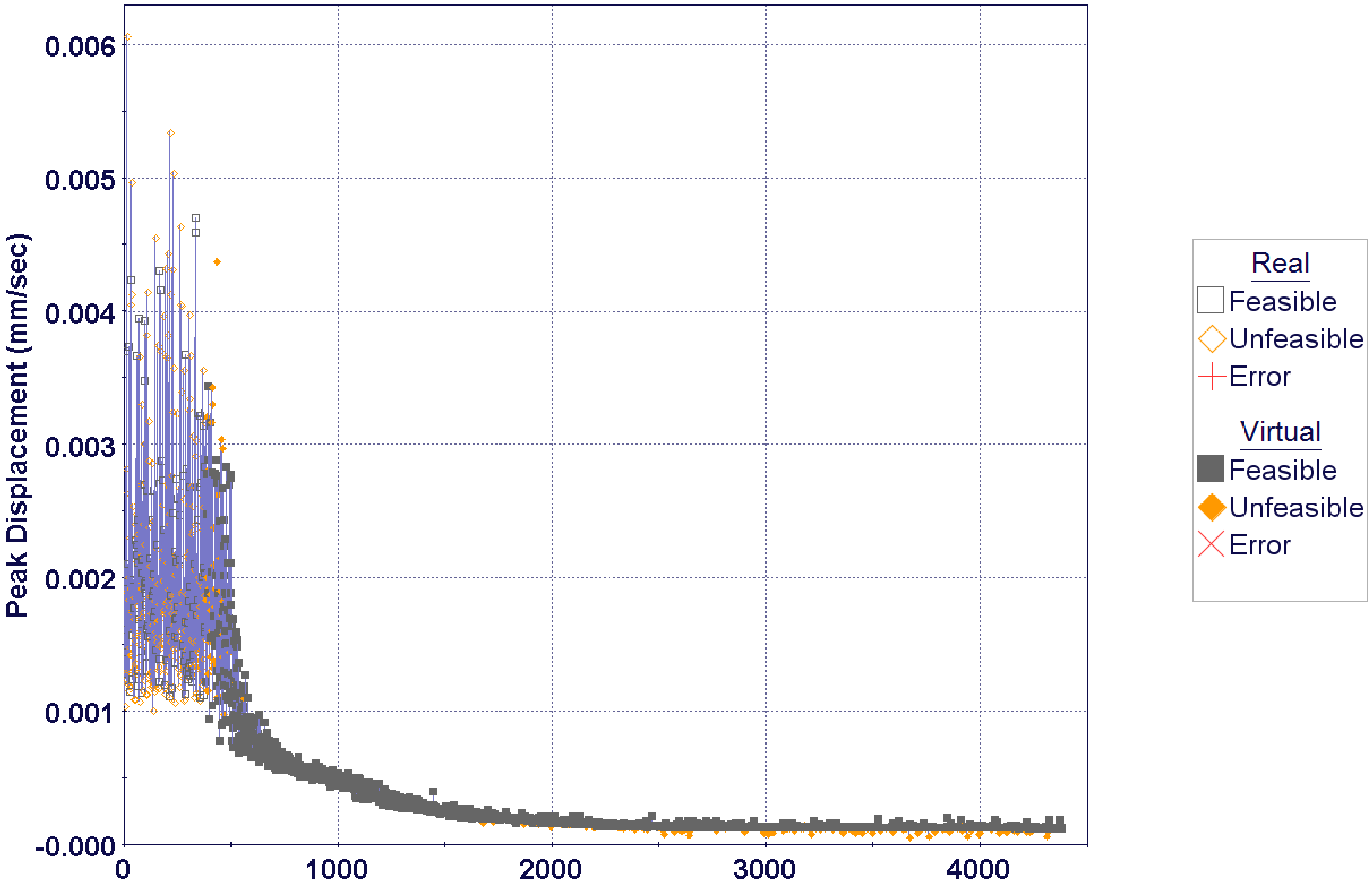
- Kriging Metamodel (RSM)
- Design of Experiments using incremental space filling algorithm

Design Pts	Average Error
40	53 %
80	35 %
120	26 %
160	31 %
200	18 %
375	10 %

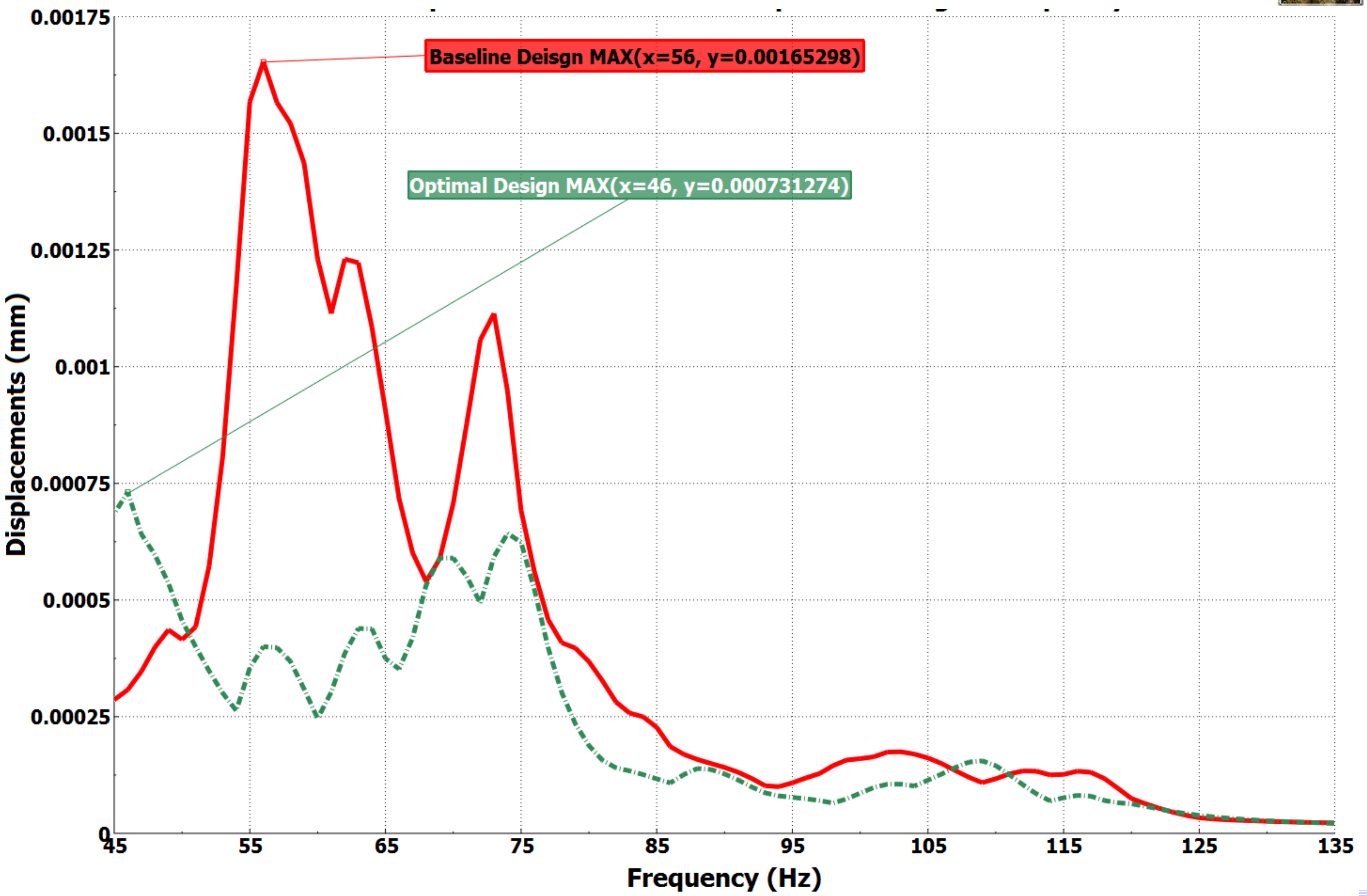
MCA Re-analysis CPU Reduction per Design Pt	3 min 21 sec
375 Designs	21.1 hrs

ModeFRONTIER was used

Optimization History



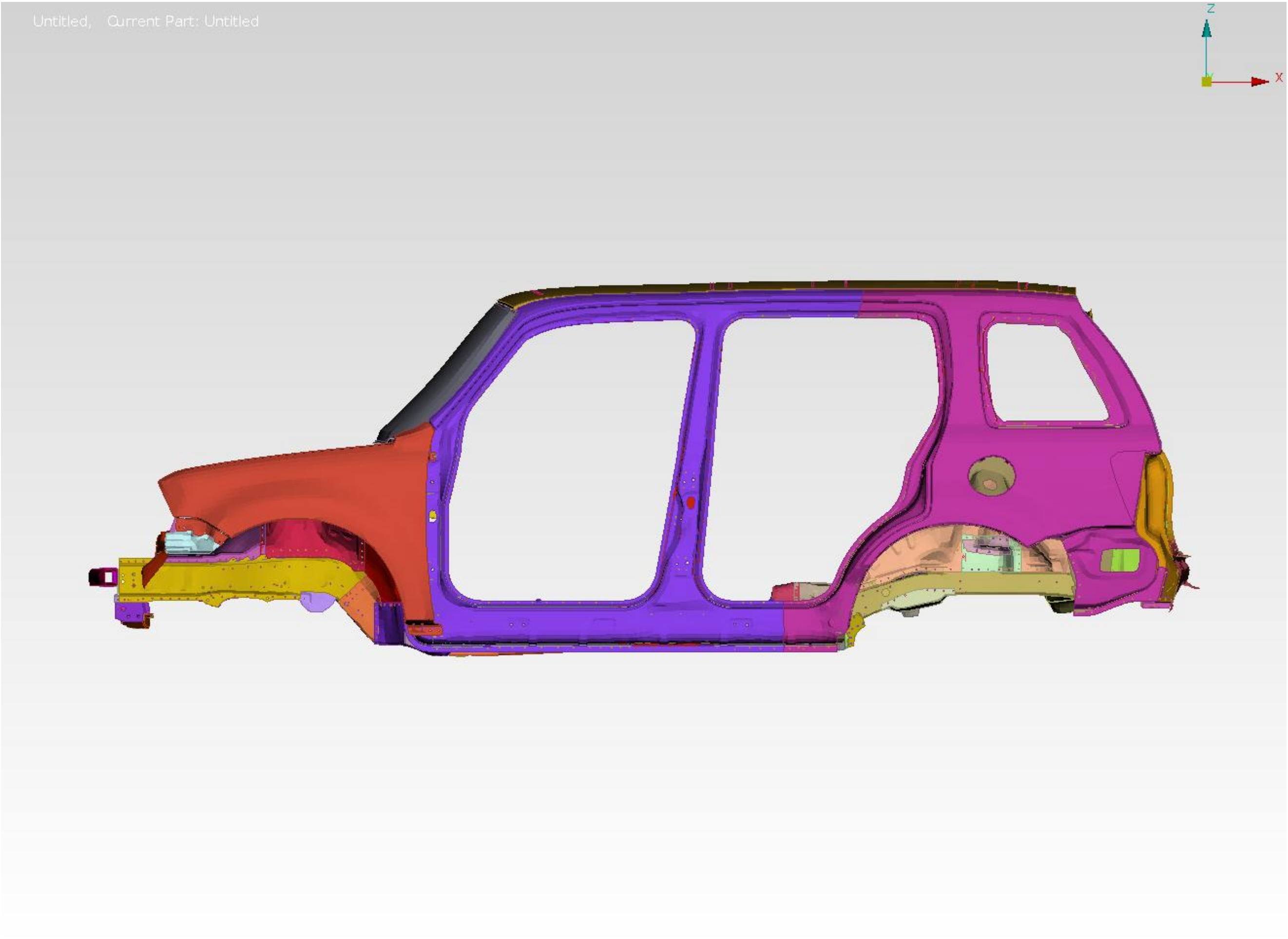
Baseline and Optimal Designs



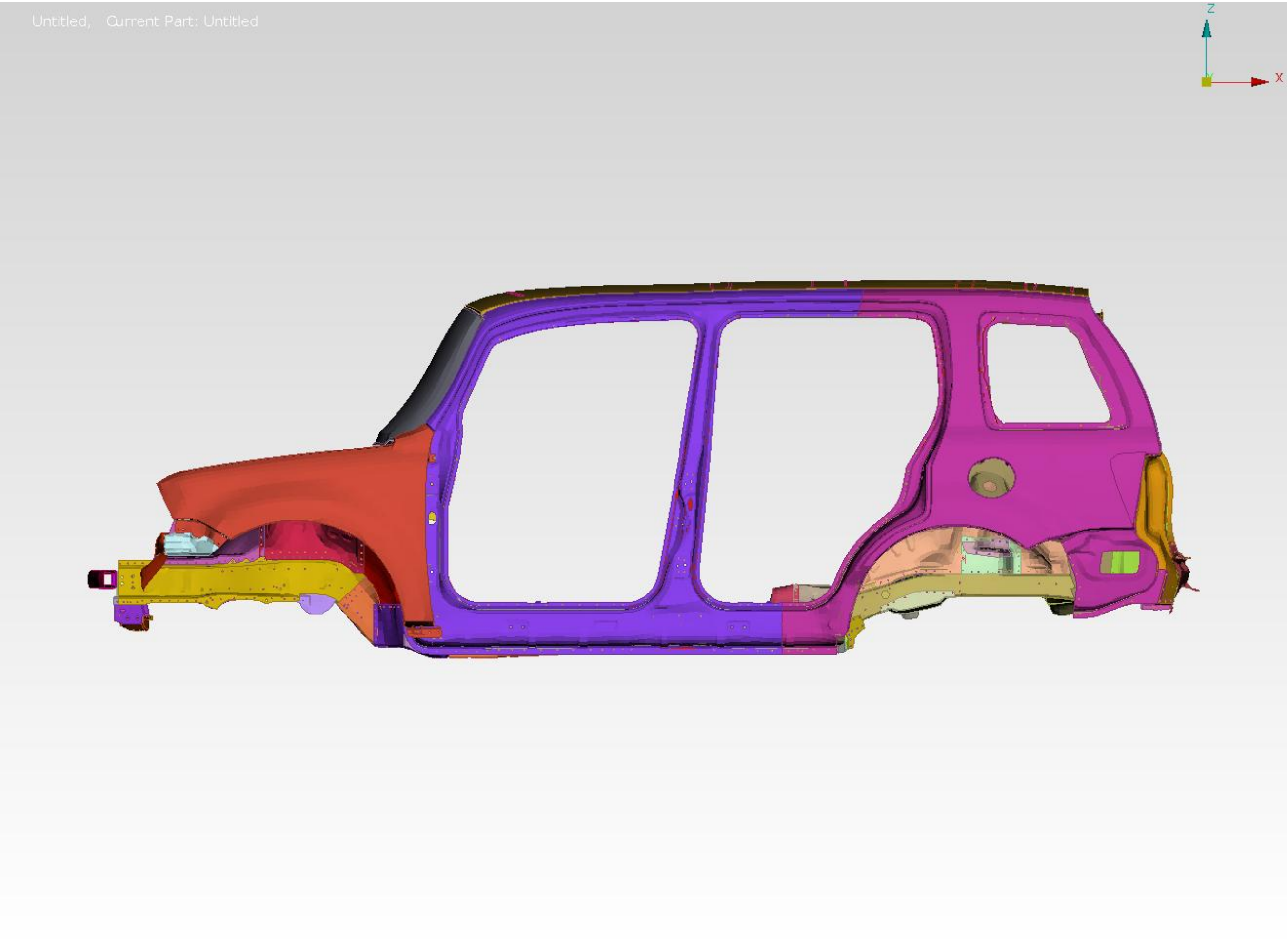
Baseline and Optimal Values

No.	Parameter Name	Baseline	Optimal	Lower Bound (mm)	Upper Bound (mm)
1	BPillar_Width	0	-9.92708	-10	40
2	RR Header X Position	0	54.24584	-150	150
3	Windshield Header Position	0	-129.999	-130	130
4	Roof Height	0	6.298661	-130	150
5	7850276 Cross Bow 1	0.75	0.732604	0.4	2
6	7850263 Cross Bow Main	0.75	1.999987	0.4	2
7	7824312_ B-Pillar Inner RT Gauge	1.2	0.517935	0.4	2
8	78502076_Roof Cross Bow 2	0.75	1.165611	0.4	2
9	7824557_B-Pillar Inner RT Gauge	1.6	0.400019	0.4	2
10	47071 Floor Cross Member Rea	1.4	1.387041	0.4	2
11	7810628 Floor Seat Cross Member	0.75	1.079494	0.4	2
12	92 Roof Cross Bow 3	0.75	1.999551	0.4	2
13	7850202 Roof Gauge	0.79	1.999617	0.4	2

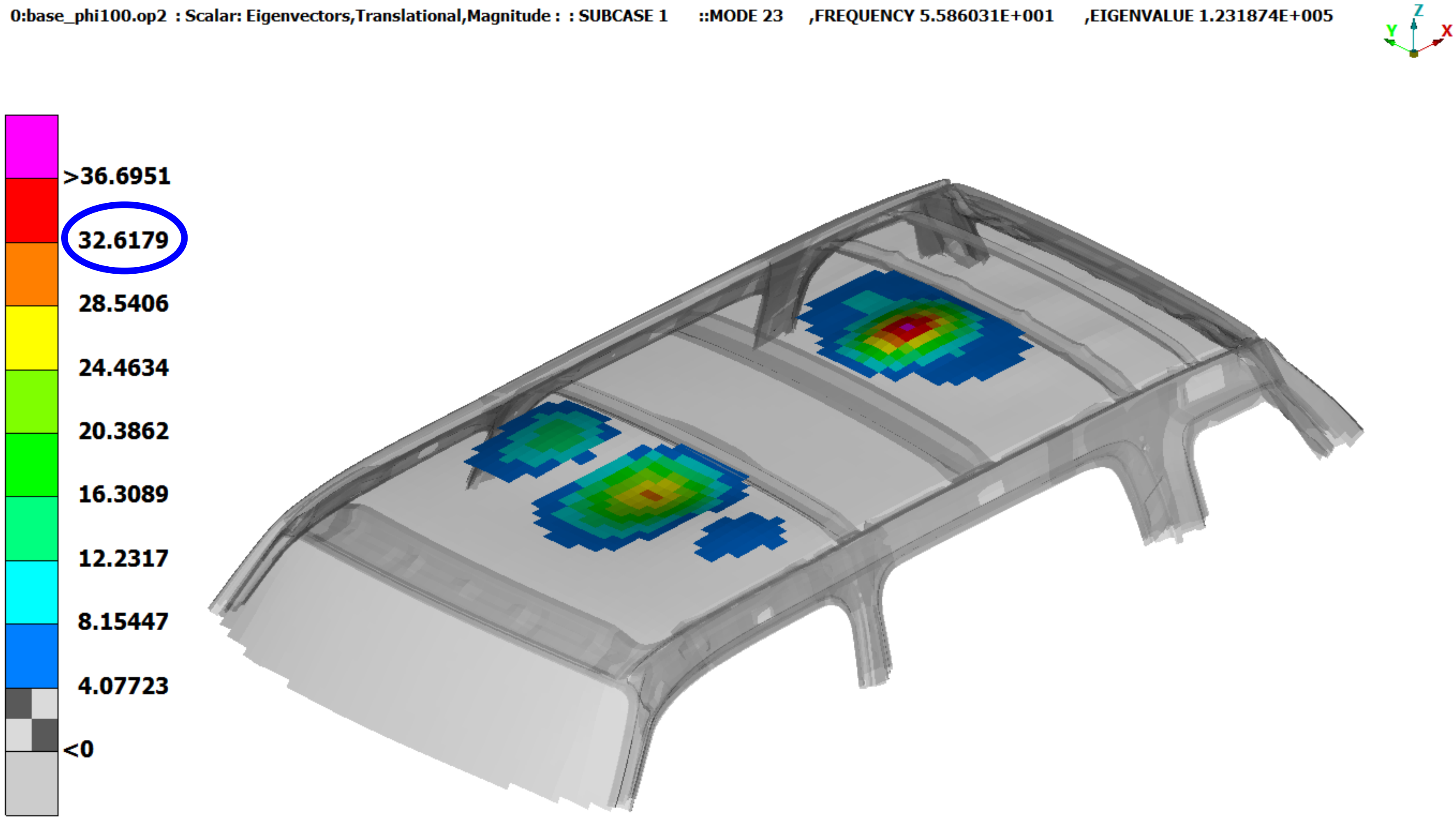
Baseline Design



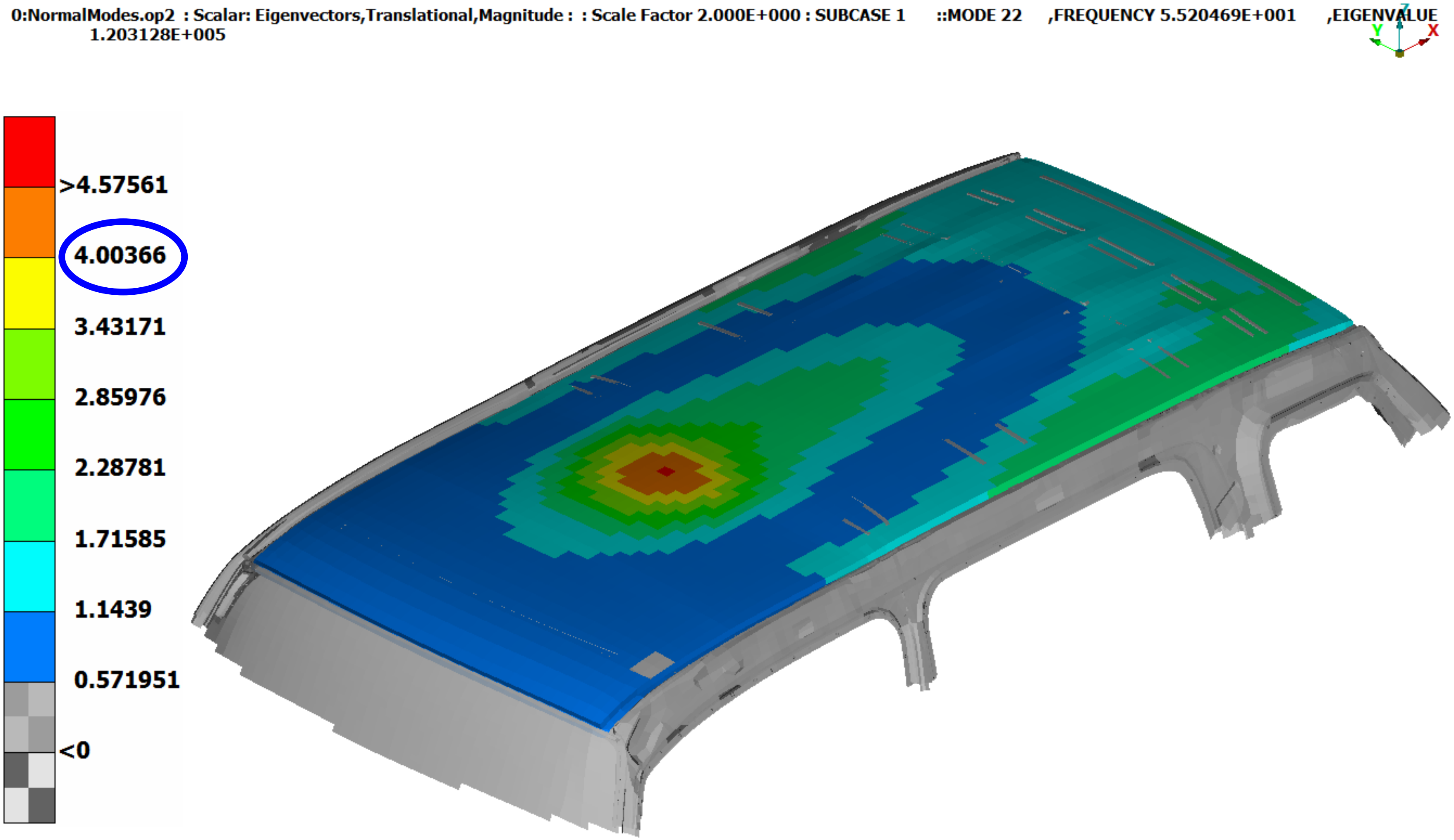
Optimal Design



Baseline : 55.8 Hz Mode



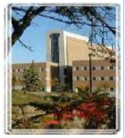
Optimal : 55.2 Hz Mode

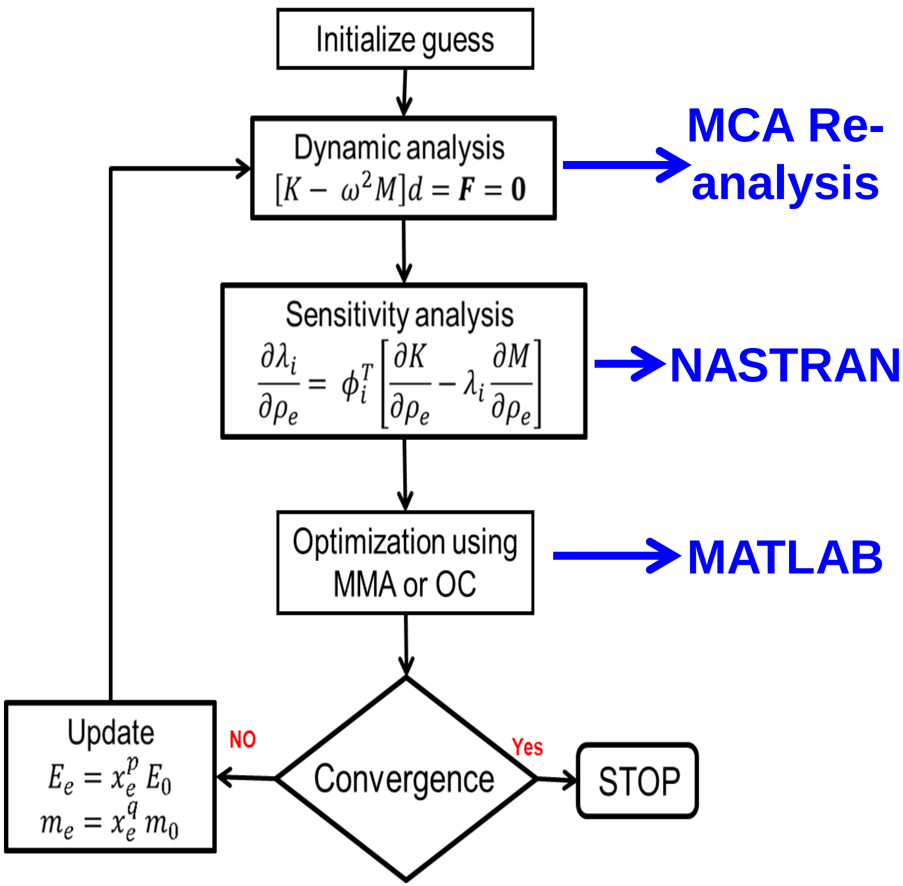


Topology Optimization with Re-Analysis Methods

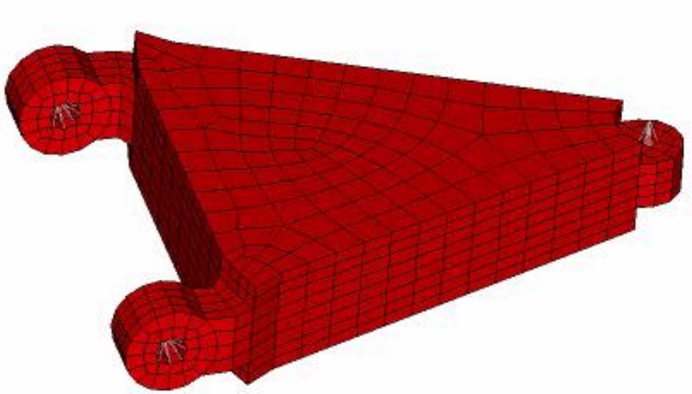
- Optimizer uses MMA - Method of Moving Asymptotes (in MATLAB)
  - Gradient Filtering to reduce mesh dependency
  - Paulson Scheme to avoid discontinuity / checker board
- NASTRAN for all calculations (mass, volume, objective, constraints, AND gradients)
- MCA Re-analysis for all eigenvalue / eigenvector estimations

Topology Optimization with Re-analysis





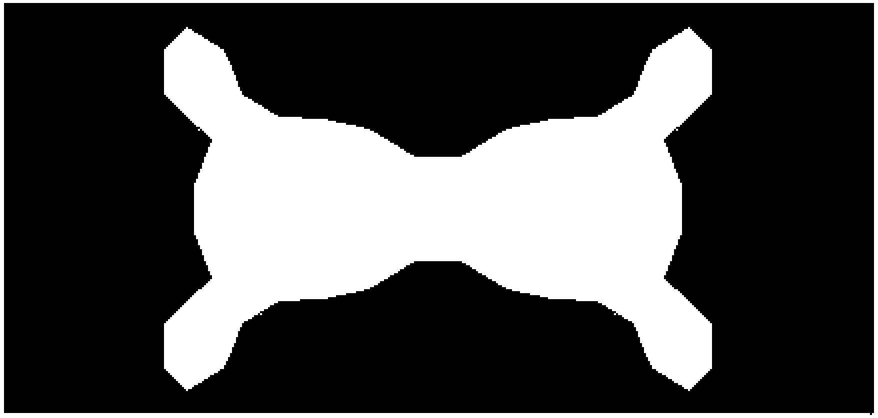
$$
\begin{aligned}
&\max\{\ \lambda_i\}, && i = 1, \dots\dots, N_{dof} \\
&s.t.\ (K - \lambda_i M)\phi_i = 0, && i = 1, \dots\dots, N_{dof} \\
&\textstyle\sum_{e=1}^{N} v_e \rho_e \le V, && 0 < \rho_{min} < \rho_e < 1, \\
& && e = 1, \dots., N
\end{aligned}
$$



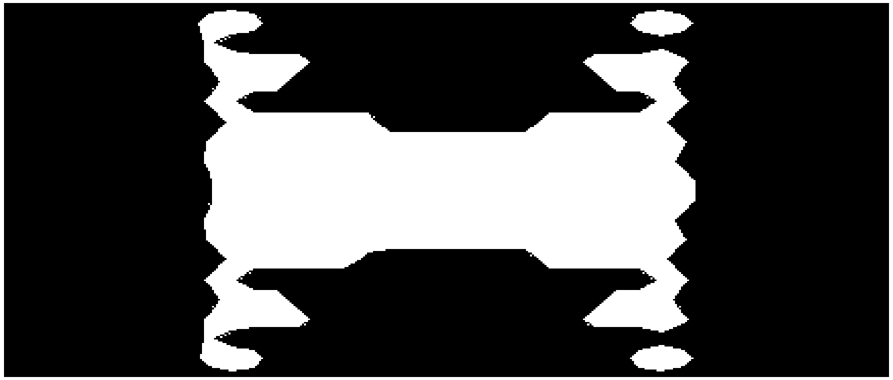
Gradient Filtering for Mesh Dependency

Fine mesh provides more detail which can result in localized small holes

20 x 10 coarse mesh

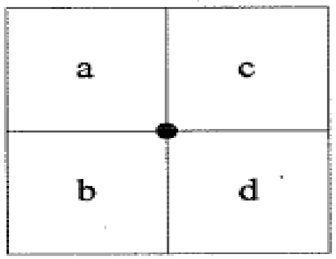


40 x 20 fine mesh

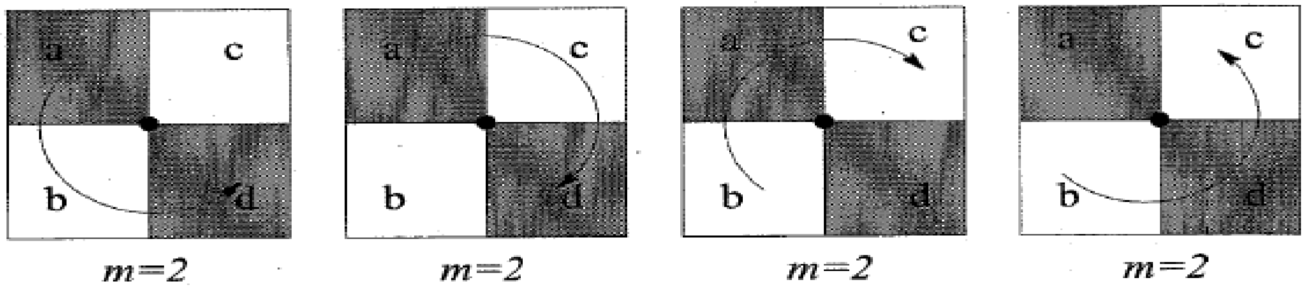


Paulson Scheme

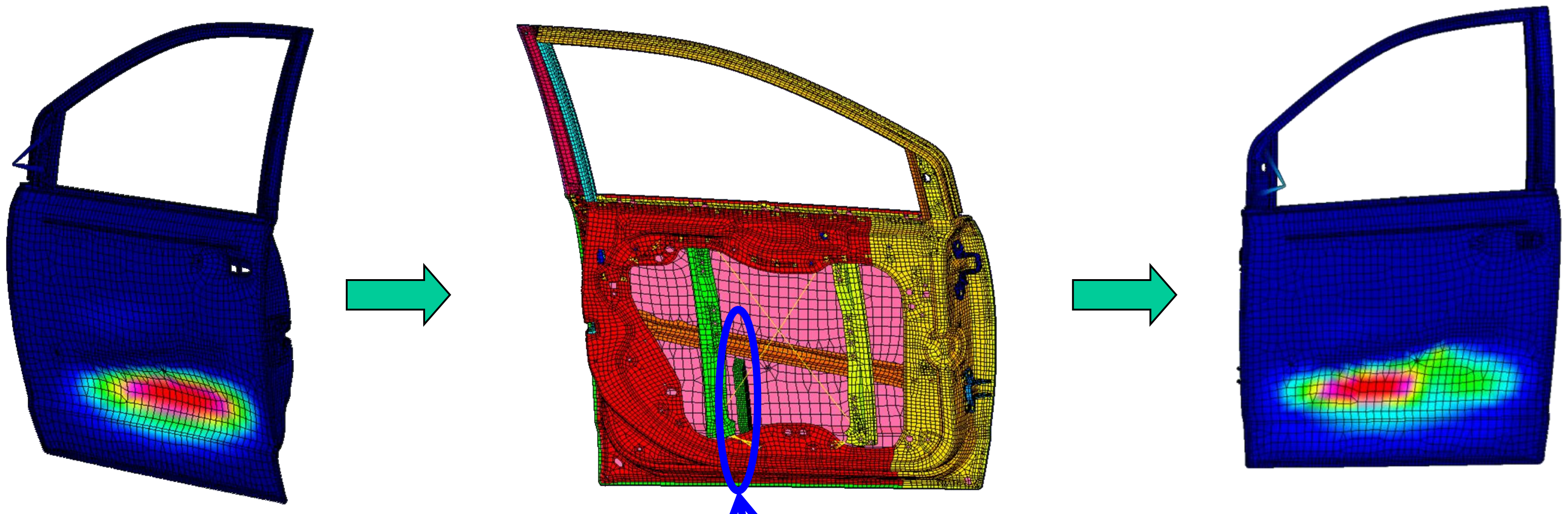
Prevents checker-board pattern producing one-node hinge connection



Four-element 2D example



Automotive Door Example



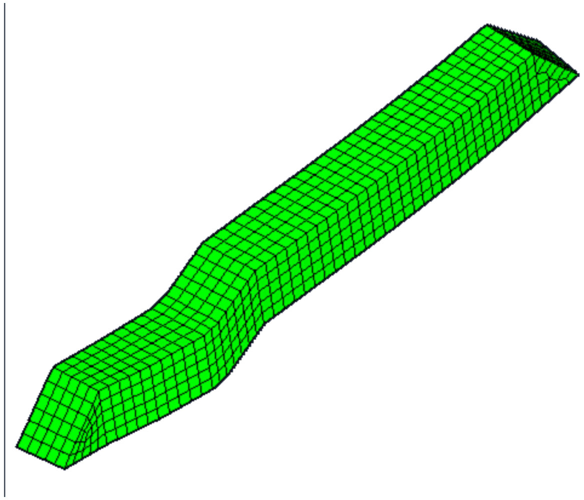
Original Model :
32.2 Hz Mode

Added Stiffener

Modified Model :
44.9 Hz Mode

Automotive Door Example

Stiffener Details and Optimization Problem



848 CTRIA3 and CQUAD4 elements (thickness of all elements are design variables)

Optimization Problem:

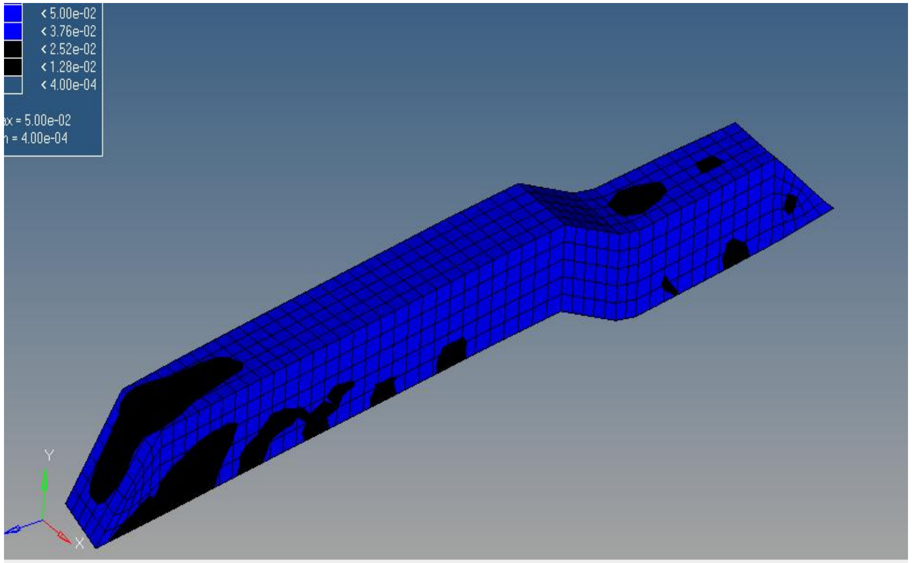
Maximize frequency such that:

Mass/volume of stiffener is less than 80% of original mass/volume.

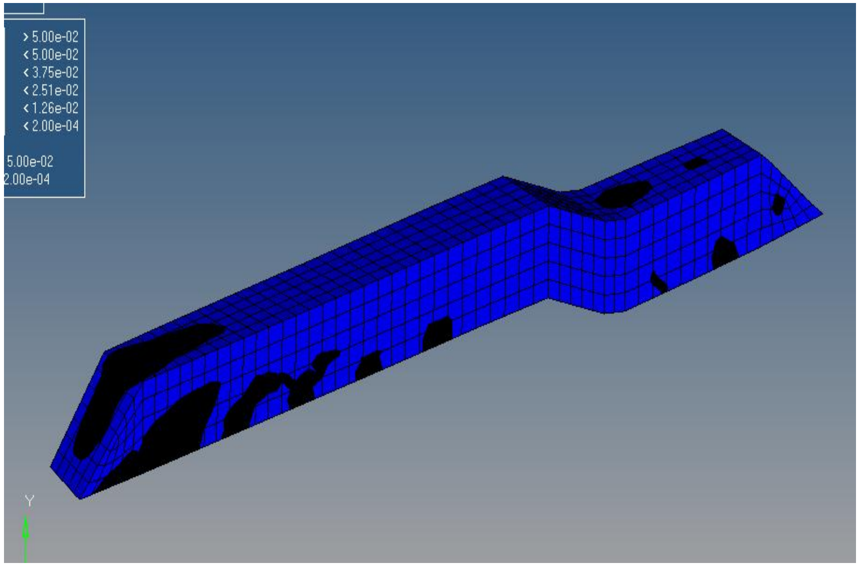
Automotive Door Example

Topology Optimal Design

Gradient by Nastran



Gradient by Nastran+MCA



Using MCA reduced computational effort from 3 hours to 1 hour

Summary and Conclusions

- **Re-Analysis methods for gauge, shape and topology changes can be very effective for problems with repetitive analyses such as optimization.**
  - **CDH method is not recommended for shape and topology changes**
  - **MCA method is recommended for shape and topology changes**
- **Current research concentrates on refining re-analysis methods for shape, topology, and probabilistic optimization**

Thanks for your Attention !!

Q & A

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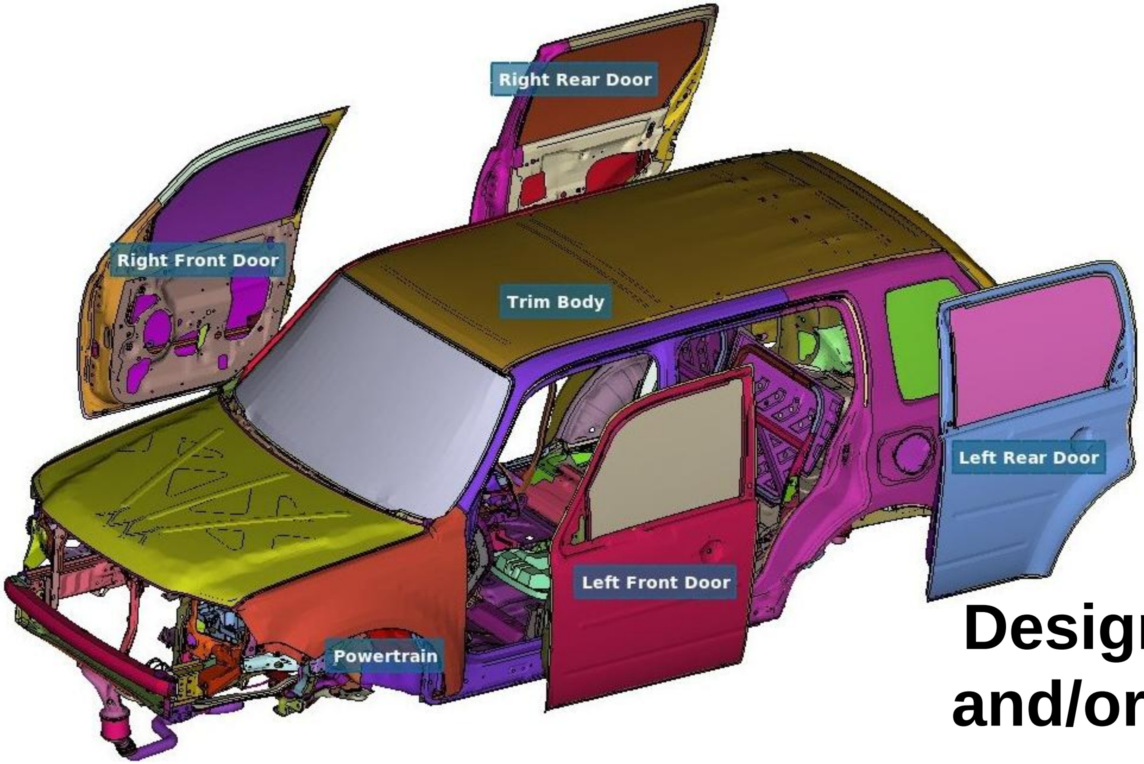
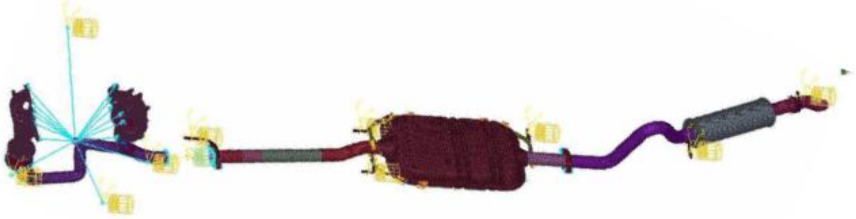
FRF Substructuring





Design changes on doors and/or powertrain-exhaust

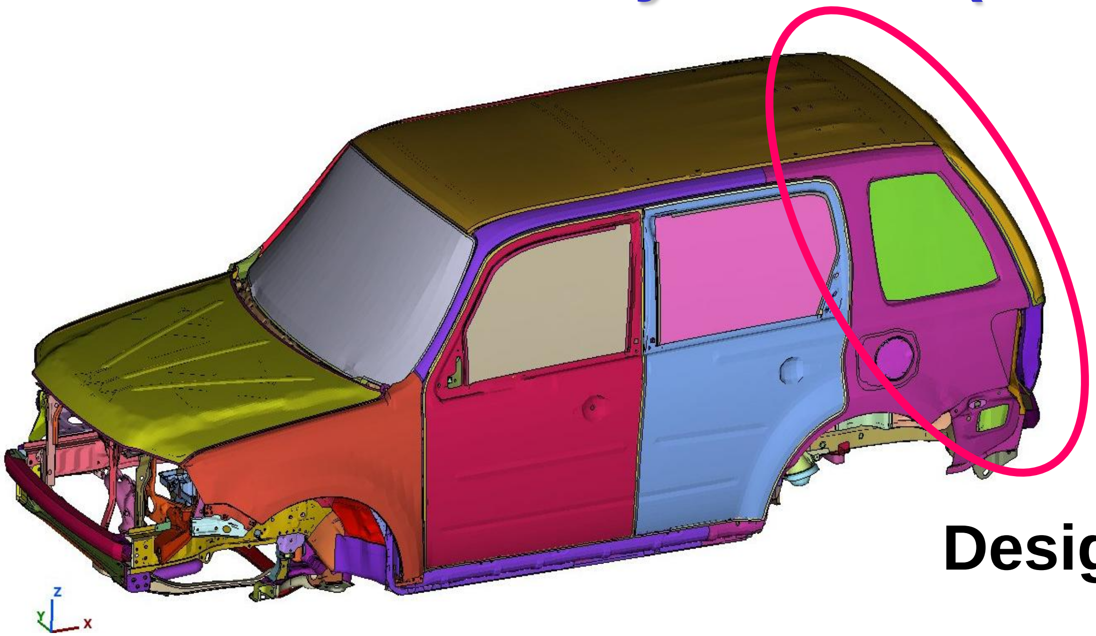
⇩

Few interface DOFs

⇩

FRF substructuring

Craig-Bampton Component Mode Synthesis (CMS)



Design changes on rear of vehicle

⇓

Many interface DOFs

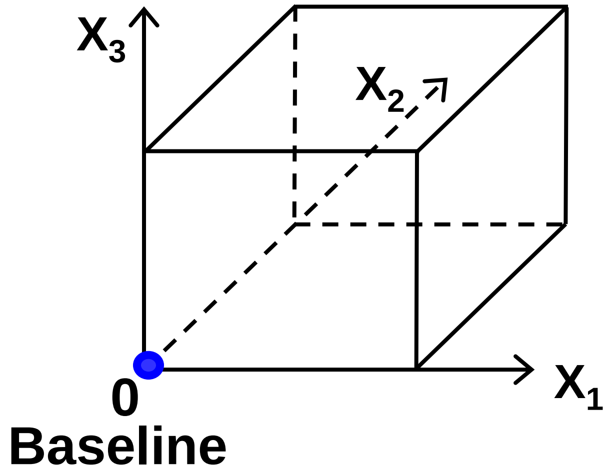
⇓

CMS (Component Mode Synthesis)

Overview / Demonstration of Re-Analysis Methods

- **CDH/VBA, MCA and PROM Re-Analysis Methods**
- **Simplified Triple Product**
  - **Polynomial Regression Method**
  - **Large Perturbation Response using Successive Iterations**
- **Re-Analysis Methods for Shape Changes**
  - **Different Examples**
  - **Shape/Gauge Optimization (Car Example)**
- **Re-Analysis in Topology Optimization**
  - **Automotive Door Example**

CDH/VBA Re-Analysis Method



$$\mathbf{K}_R = \mathbf{\Phi}_0^T \mathbf{K} \; \mathbf{\Phi}_0 \qquad \mathbf{M}_R = \mathbf{\Phi}_0^T \mathbf{M} \; \mathbf{\Phi}_0$$

Reduced Matrices

Modal Basis at Baseline

Modified Combined Approximation (MCA) Re-Analysis Method

Direct Solution of Modified Eigenproblem

$$\mathbf{K}\,\boldsymbol{\Phi} = \lambda\,\mathbf{M}\,\boldsymbol{\Phi}$$

or $$\boldsymbol{\Phi} = \lambda\,\mathbf{K}^{-1}\mathbf{M}\,\boldsymbol{\Phi}$$

Iterative Solution of Modified Eigenproblem

$$\boldsymbol{\Phi}_j = \lambda\,\mathbf{K}^{-1}\mathbf{M}\,\boldsymbol{\Phi}_{j-1}$$

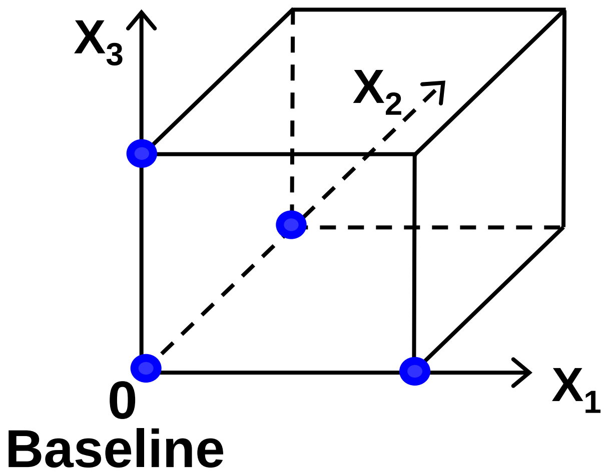
Definition and formation of basis vectors

$$\mathbf{T}_1 = \mathbf{K}^{-1}\left(\mathbf{M}\,\boldsymbol{\Phi}_0\right)$$

$$\mathbf{T}_i = \mathbf{K}^{-1}(\mathbf{M}\,\mathbf{T}_{i-1}) \qquad i = 2,3,\cdots,s$$

$$\mathbf{T} = \left[\boldsymbol{\Phi}_0 \quad \mathbf{T}_1 \quad \mathbf{T}_2 \quad \cdots \quad \mathbf{T}_s\right]$$

PROM: Parametric Reduced Order Modeling



X_1,X_2,X_3: design variables

Eigen-analysis of full matrices

$$eig(\mathbf{K}\ ,\mathbf{M}\ ) \Rightarrow \lambda\ ,\mathbf{\Phi}$$

1. Form reduced basis:

$$\mathbf{P} = [\mathbf{\Phi}_0 \quad \mathbf{\Phi}_1 \quad \mathbf{\Phi}_2 \quad \mathbf{\Phi}_3]$$

2. Project system matrices to reduced basis:

$$\mathbf{K}_R = \mathbf{P}^T \mathbf{K}\ \mathbf{P} \qquad \mathbf{M}_R = \mathbf{P}^T \mathbf{M}\ \mathbf{P}$$

3. Eigen-analysis of reduced matrices:

$$eig(\mathbf{K}_R, \mathbf{M}_R) \Rightarrow \tilde{\lambda}^p, \mathbf{\Theta}$$

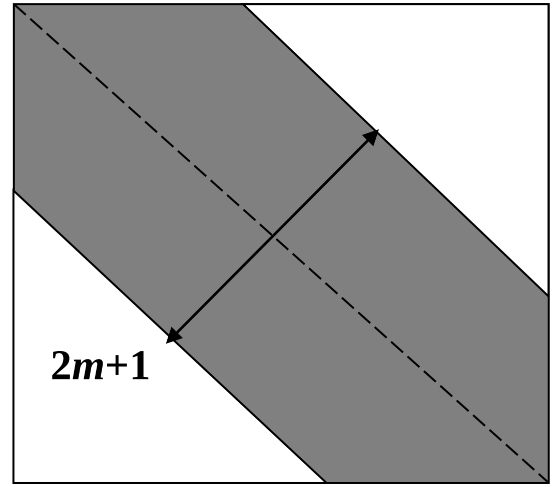
4. Obtain approximate eigenvectors:

$$\tilde{\mathbf{\Phi}}\ = \mathbf{P}\ \mathbf{\Theta}$$

- **Advantage**: Re-analysis is done using the reduced basis
- **Disadvantage**: Condensation of matrices can be expensive due to large size and high density of mode matrix P

Simplified Triple Product

100% density for condensed K and M



Density is :

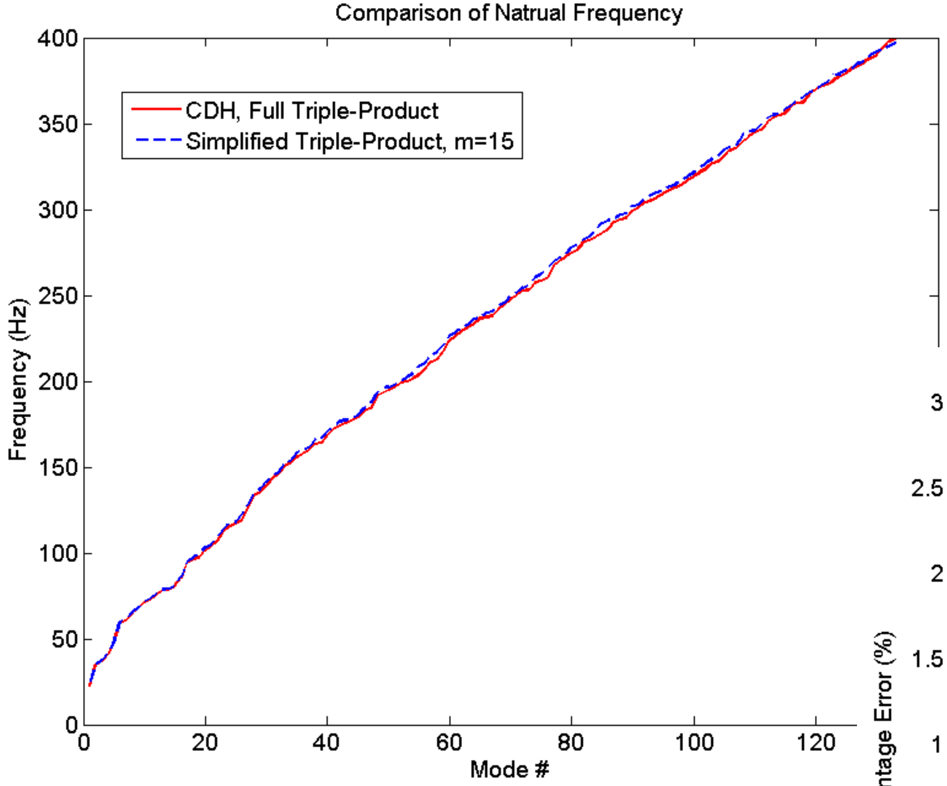
$$\frac{(2m+1)n-(m+1)m}{n^2} \approx \frac{2m+1}{n}$$

$$n >> m$$

n : Number of DOFs

Computational cost is approximately equal to $(2m+1)/n$ of full triple-product cost

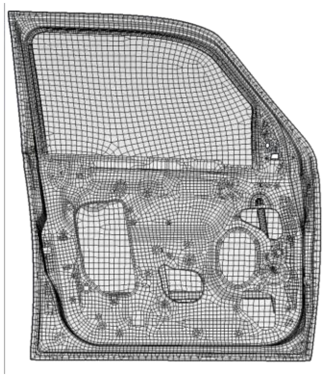
Simplified Triple Product

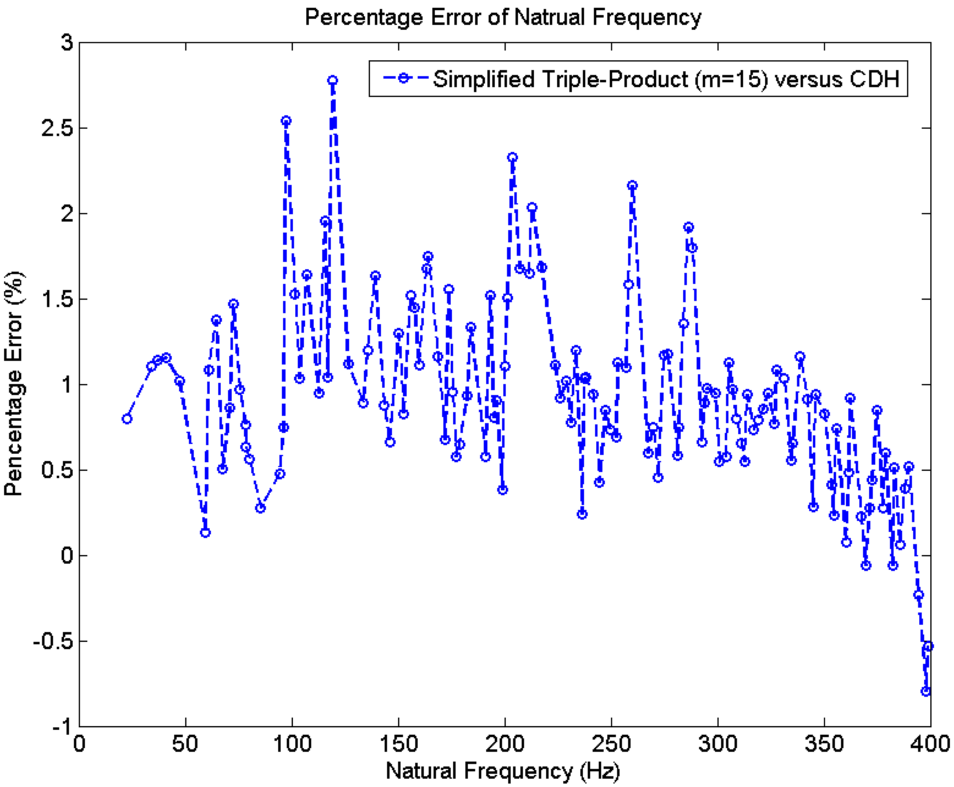


m = 15

Density : 13.1 %

20% increase of outer shell thickness





Polynomial Regression for Triple Product

Example: Thin shell elements without transverse shear

$$\mathbf{K} = \mathbf{A}_0 + \mathbf{A}_1 t + \mathbf{A}_2 t^3$$

$\mathbf{A}_i,\ i = 0,1,2$ **are fixed matrices**

Shell thickness

Similarly :

$$\mathbf{K}^r = \mathbf{A}^r_0 + \mathbf{A}^r_1 t + \mathbf{A}^r_2 t^3$$

with $\mathbf{A}^r_i = \mathbf{\Phi}^T \mathbf{A}_i \mathbf{\Phi}$

Oakland University

Polynomial Regression for Triple Product

Calculate triple products for 3 different thicknesses :

$$\begin{bmatrix} 1.0 & t_0 & t_0^3 \\ 1.0 & t_1 & t_1^3 \\ 1.0 & t_2 & t_2^3 \end{bmatrix} \begin{bmatrix} \mathbf{A}_0^r \\ \mathbf{A}_1^r \\ \mathbf{A}_2^r \end{bmatrix} = \begin{bmatrix} \mathbf{K}_0^r \\ \mathbf{K}_1^r \\ \mathbf{K}_2^r \end{bmatrix} \Rightarrow \mathbf{A}_i^r, \quad i = 0,1,2$$

Then:

$$\mathbf{K}^r = \mathbf{A}_0^r + \mathbf{A}_1^r t + \mathbf{A}_2^r t^3$$

Similar expressions exist for K and M matrices of plate and solid elements.

Large Perturbation Response using Successive Iterations

1. Calculate the **baseline** eigenvectors Φ_0.
2. **Perturb design variables by 5%** or so. Use **Polynomial Regression approach** to calculate reduced matrices

$$\mathbf{K}_r^1 = \Phi_0^T \mathbf{K}_1 \Phi_0 \quad \text{and} \quad \mathbf{M}_r^1 = \Phi_0^T \mathbf{M}_1 \Phi_0$$

and solve eigenvalue problem $\mathbf{K}_r^1 \Phi_r^1 = \mathbf{M}_r^1 \Phi_r^1 \Lambda_r$ for eigenvectors Φ_r^1. The eigenvectors of the perturbed structure are equal to $\Phi_1 = \Phi_o \Phi_r^1$.

3. Repeat step 2 until the maximum value of all design variables is reached. The following **recursive process** is used for $i = 1,2,\ldots,n$:

$$\mathbf{K}_r^i = \Phi_{i-1}^T \mathbf{K}_i \Phi_{i-1} \quad \text{and} \quad \mathbf{M}_r^i = \Phi_{i-1}^T \mathbf{M}_i \Phi_{i-1}$$

$$\mathbf{K}_r^i \Phi_r^i = \mathbf{M}_r^i \Phi_r^i \Lambda_r$$

$$\Phi_i = \Phi_{i-1} \Phi_r^i$$

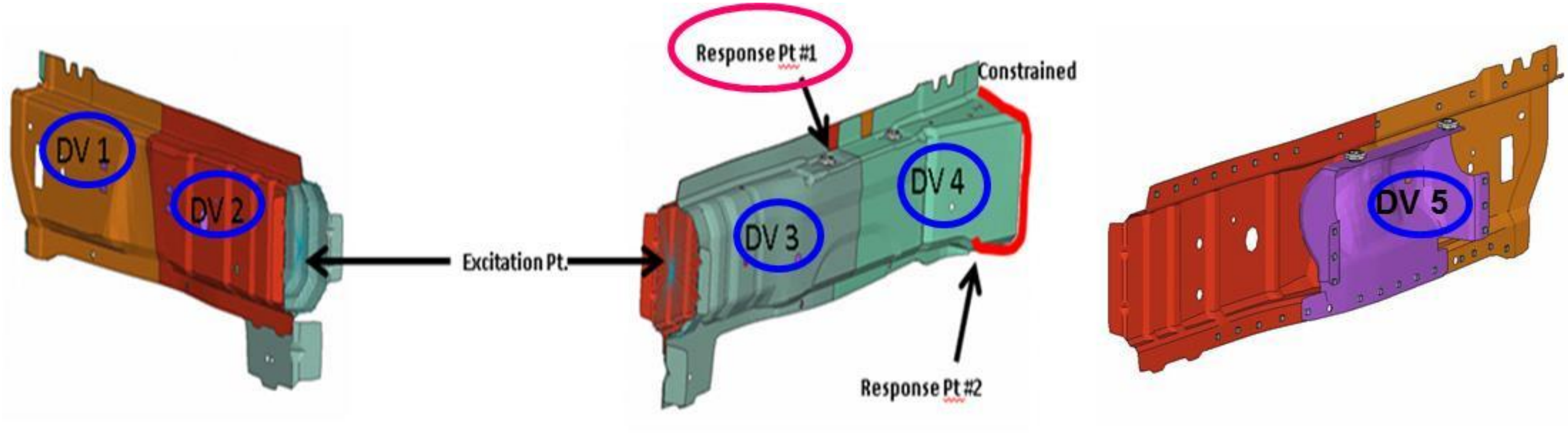
Small Size

Polynomial Regression

- No K and M matrix updates needed
- No triple product calculation is needed
- Only small eigenvalue problems are solved

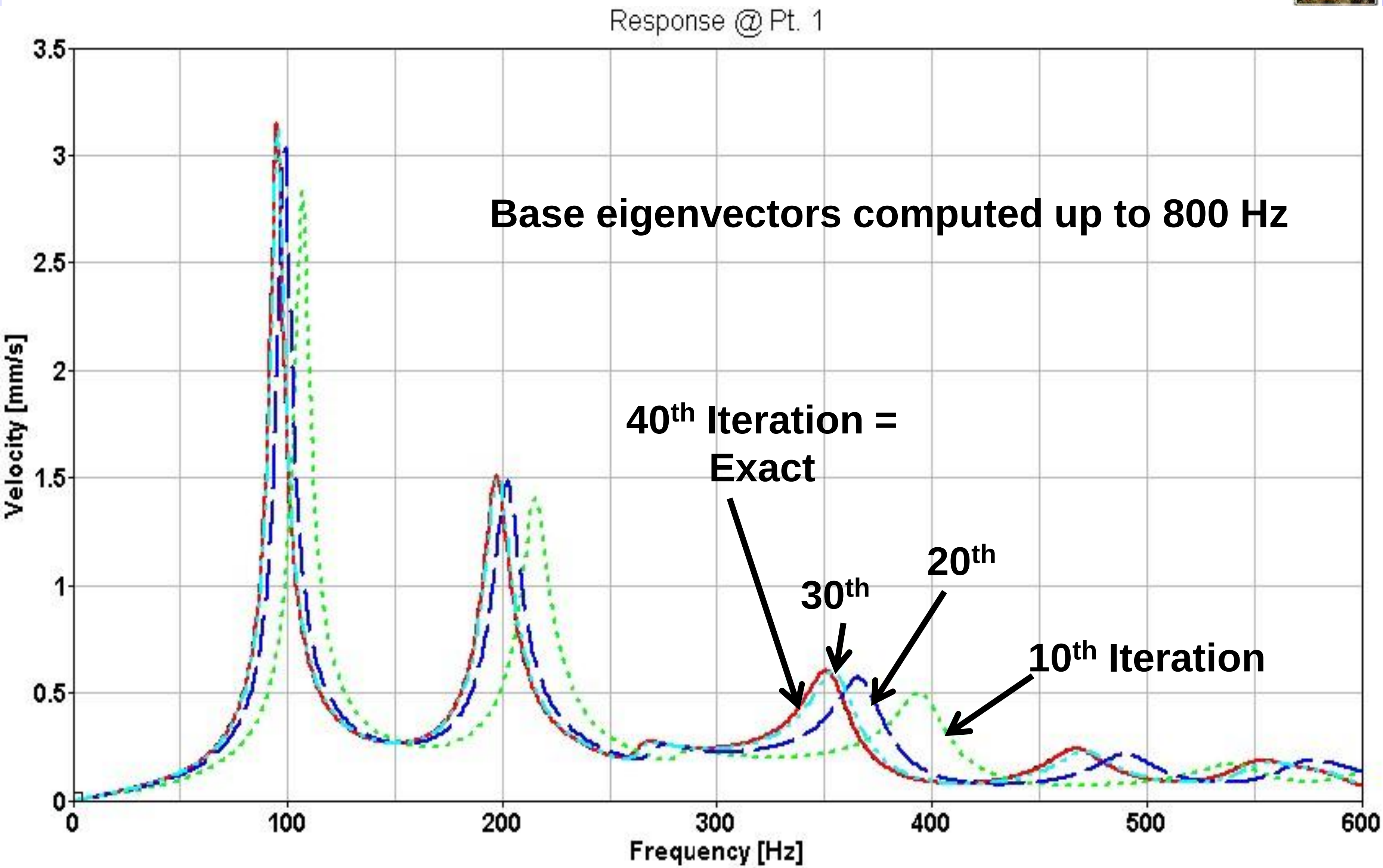
Method assumes Polynomial Regression is applicable

Automotive Rail Example

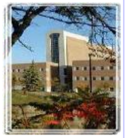


Design Variable	Min Thickness (mm)	Max Thickness (mm)	Perturbation %
1	1.025	3.075	200
2	1.16	3.48	200
3	1.25	3.75	200
4	1.16	3.48	200
5	1.25	3.75	200

Automotive Rail Example



Basis Creation / Ortho-normalization



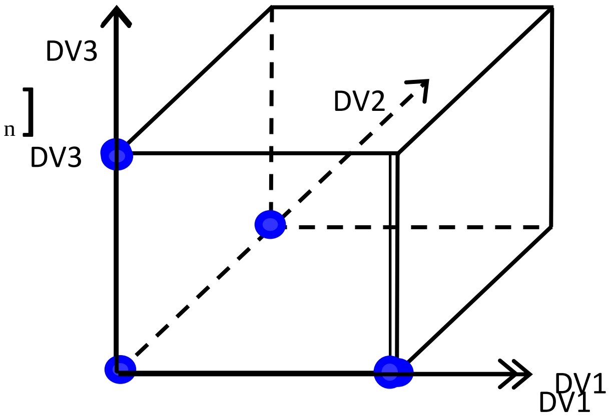
Reduction in BASIS size

Eliminate design variables

$$\Phi = [\Phi_1 \quad \cancel{\Phi_2} \quad \cancel{\Phi_3} \quad \Phi_4 \quad \cdots \quad \Phi_n]$$

$$\Downarrow$$

$$\Phi = [\Phi_1 \quad \Phi_4 \quad \cdots \quad \Phi_n]$$



Eliminate some modes

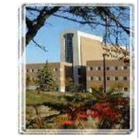
$$\Phi_i = \begin{bmatrix} \lambda_{11} & \cancel{\lambda_{12}} & \lambda_{13} & \cancel{\lambda_{14}} & \lambda_{15} & \cancel{\lambda_{16}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \lambda_{41} & \cancel{\lambda_{42}} & \lambda_{43} & \cancel{\lambda_{44}} & \lambda_{45} & \cancel{\lambda_{46}} \end{bmatrix}$$

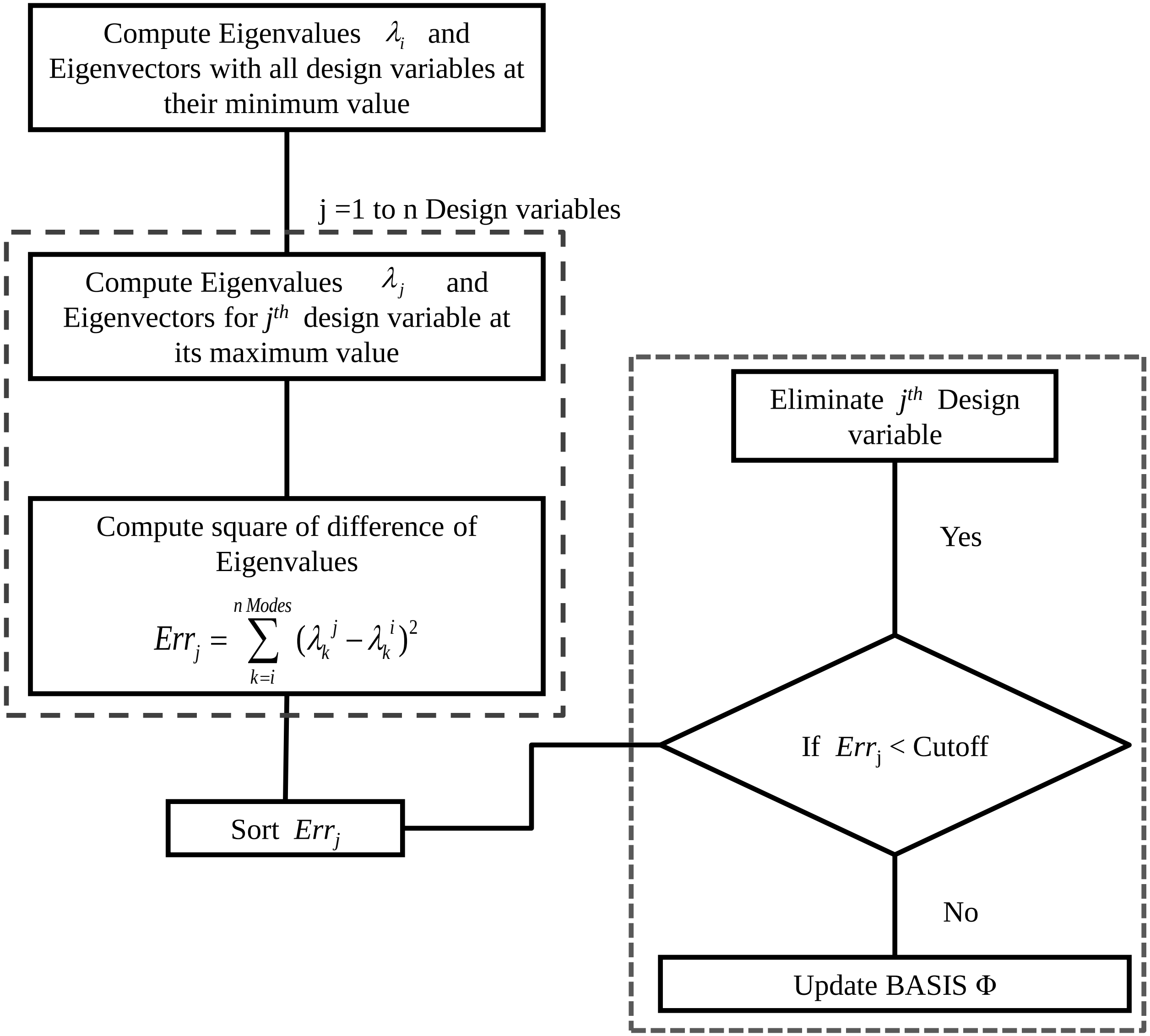
$$\Downarrow$$

$$\Phi_i = \begin{bmatrix} \lambda_{11} & \lambda_{13} & \lambda_{15} \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ \lambda_{41} & \lambda_{43} & \lambda_{45} \end{bmatrix}$$

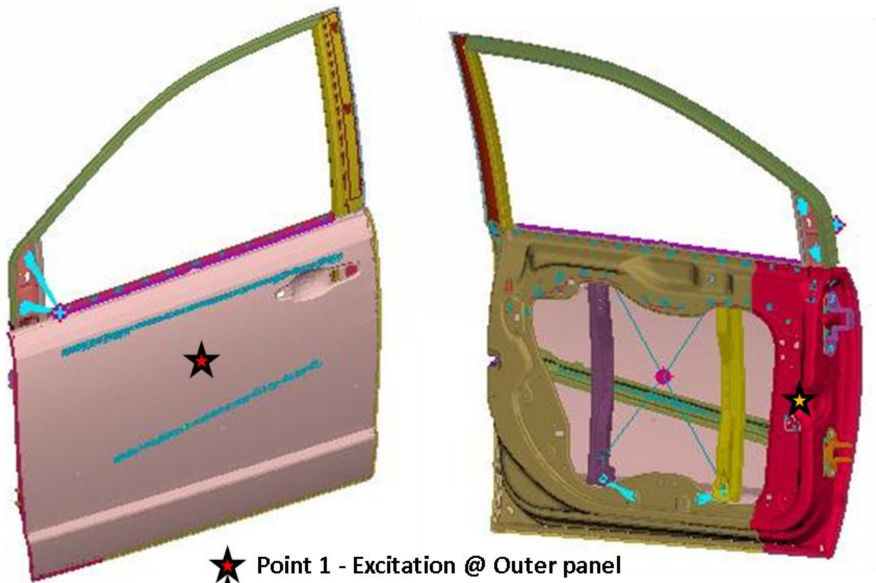
Less time to Ortho-normalize

Elimination of Design Variables





Elimination of Design Variables



Automotive door example with 1,46,589 DOF

#	Design Variable Description	Minimum	Maximum
Thickness (mm)			
1	Inner Door panel RHS	1	3
2	Door impact bar	1	2.75
3	Inner Door panel LHS	0.5	1.75
4	Upper door hinge reinforcement	1	3
5	Door surround rear	0.25	1.25
6	Outer panel	0.25	1.25
7	Door latch reinforcement	1	2
8	Door surround rear	1.25	3.25
9	Door belt inner	0.75	1.75
10	Glass channel mounting brkt.	1.25	2.25
11	Door surround rear	0.25	1.25
12	Inner/Outer panel skirt	1	3
13	Inner/Outer panel skirt	2	4
14	Door belt outer	0.25	1.25
15	Door surround glass	0.25	1.25
16	Impact bar bracket	1	3
17	Door Inner belt skirt	1	3
18	Glass runner	0.25	1
Mass (Tonne)			
19	Side view mirror mass	0.001	0.002
20	Door trim mass	0.00035	0.0005

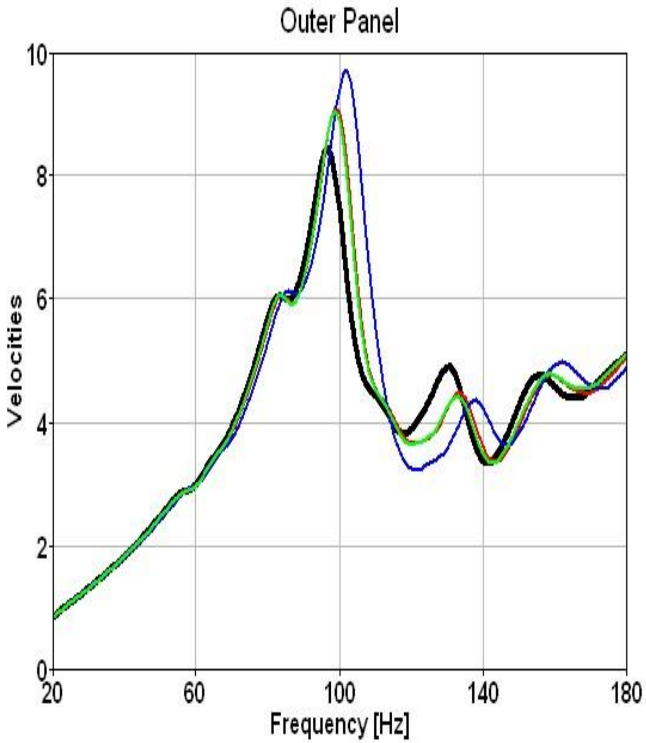
Elimination of Design Variables

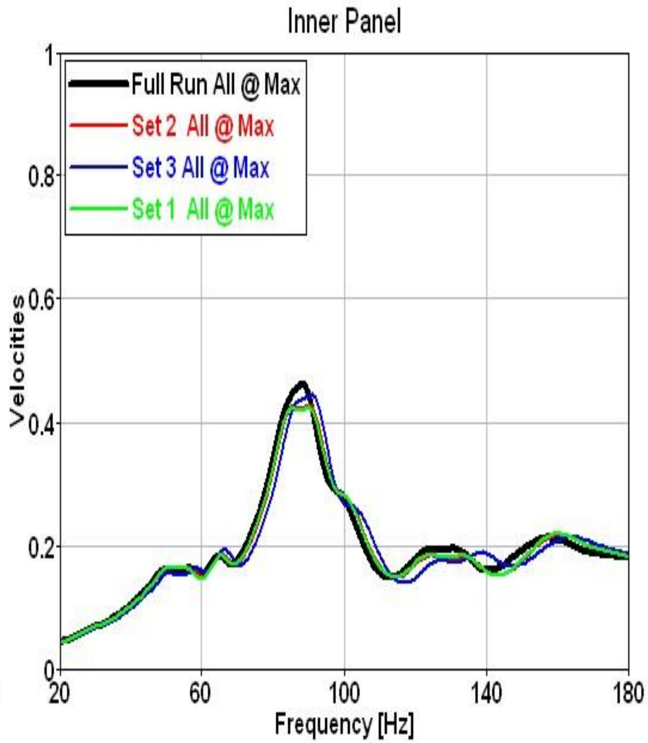
DV	Sorted Sum of Error Squared
6	370152.80
3	18147.88
14	3003.77
1	2965.82
9	867.31
15	330.78
10	151
11	121.24
5	115.34
19	115.21
18	111.67
17	105.51
2	78.83
16	73.11
7	49.56
8	49.47
12	44.23
13	10.97
20	5.53
4	0.56

3568 for all

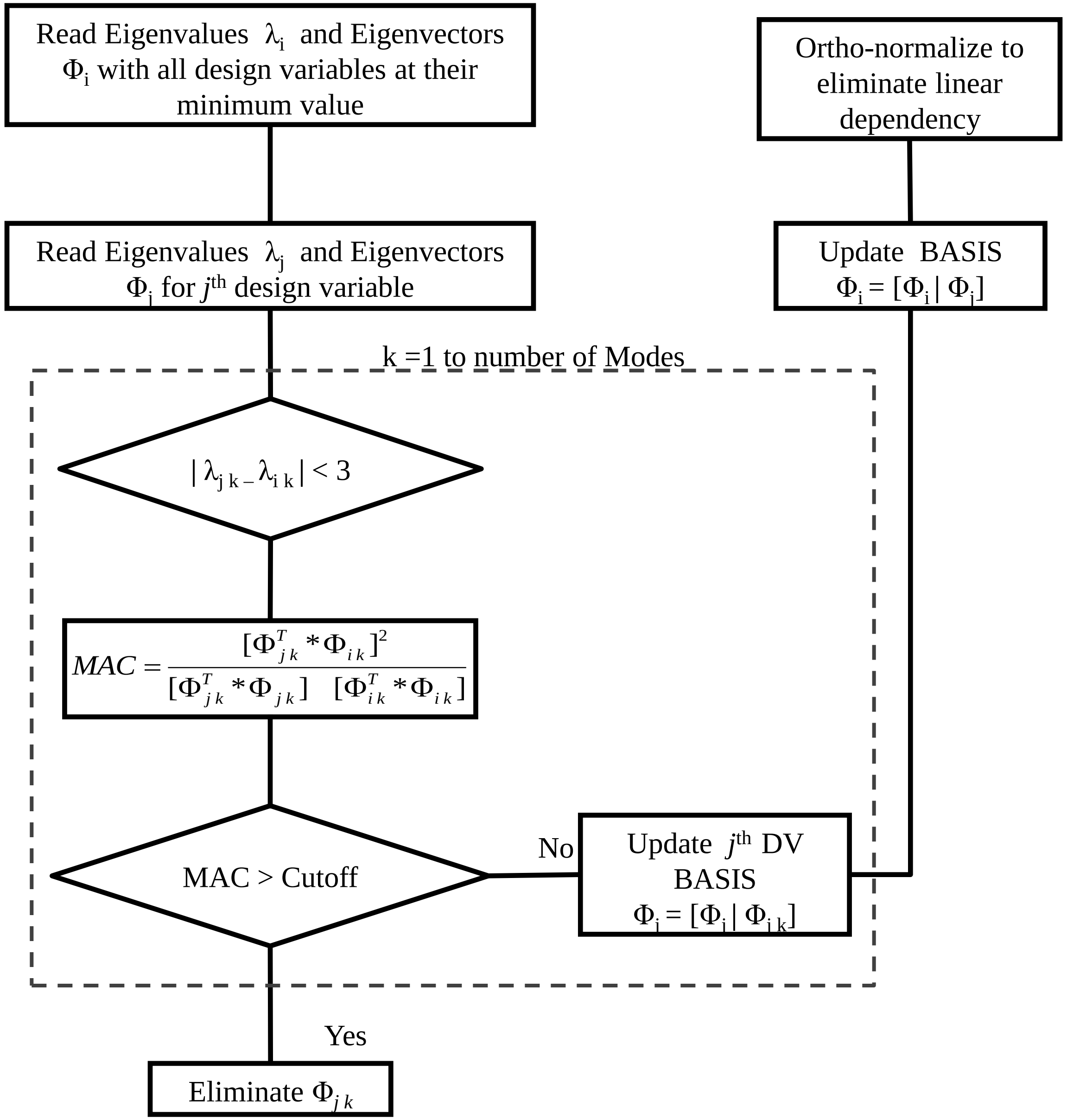
220 for all

BASIS Set #	Eliminated Design Variable	Vector columns in BASIS	Time (min) for Ortho-Normalization
1	4, 20, 13	3040	85
2	4, 20, 13, 12, 8,7	2513	72
3	4, 20, 13, 12, 8,7,16,2,18,17	1810	50



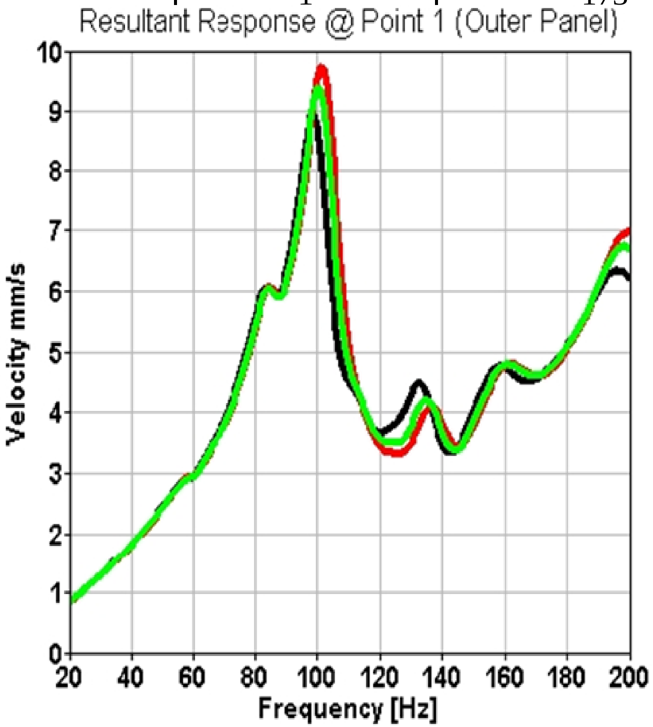


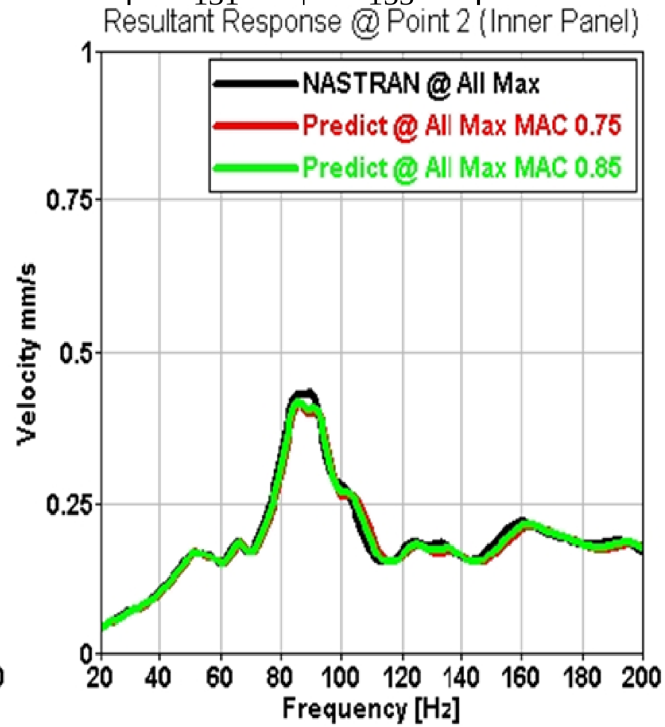
Mode Elimination



Mode Elimination

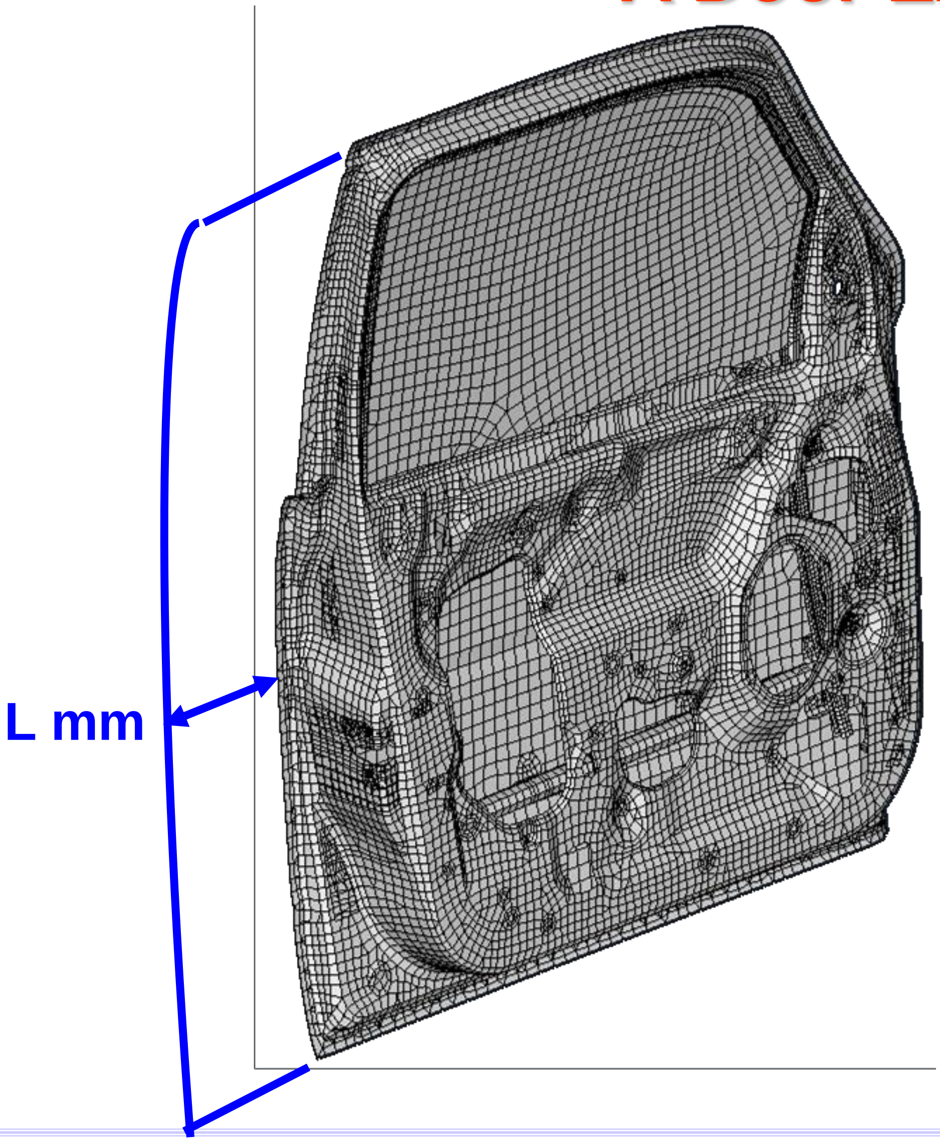
DV	No. of BASIS columns when corresponding DV is at max value only	Size Reduction MAC 0.75	MAC 0.85
BASE Model	176	176	176
1	173	131	155
Total BASIS columns	3568	1138	1478
% Reduction		68.11	58.58
Time (min.) in Ortho-Normalization and Mode elimination	220	90	75





PROM and MCA for Shape Changes

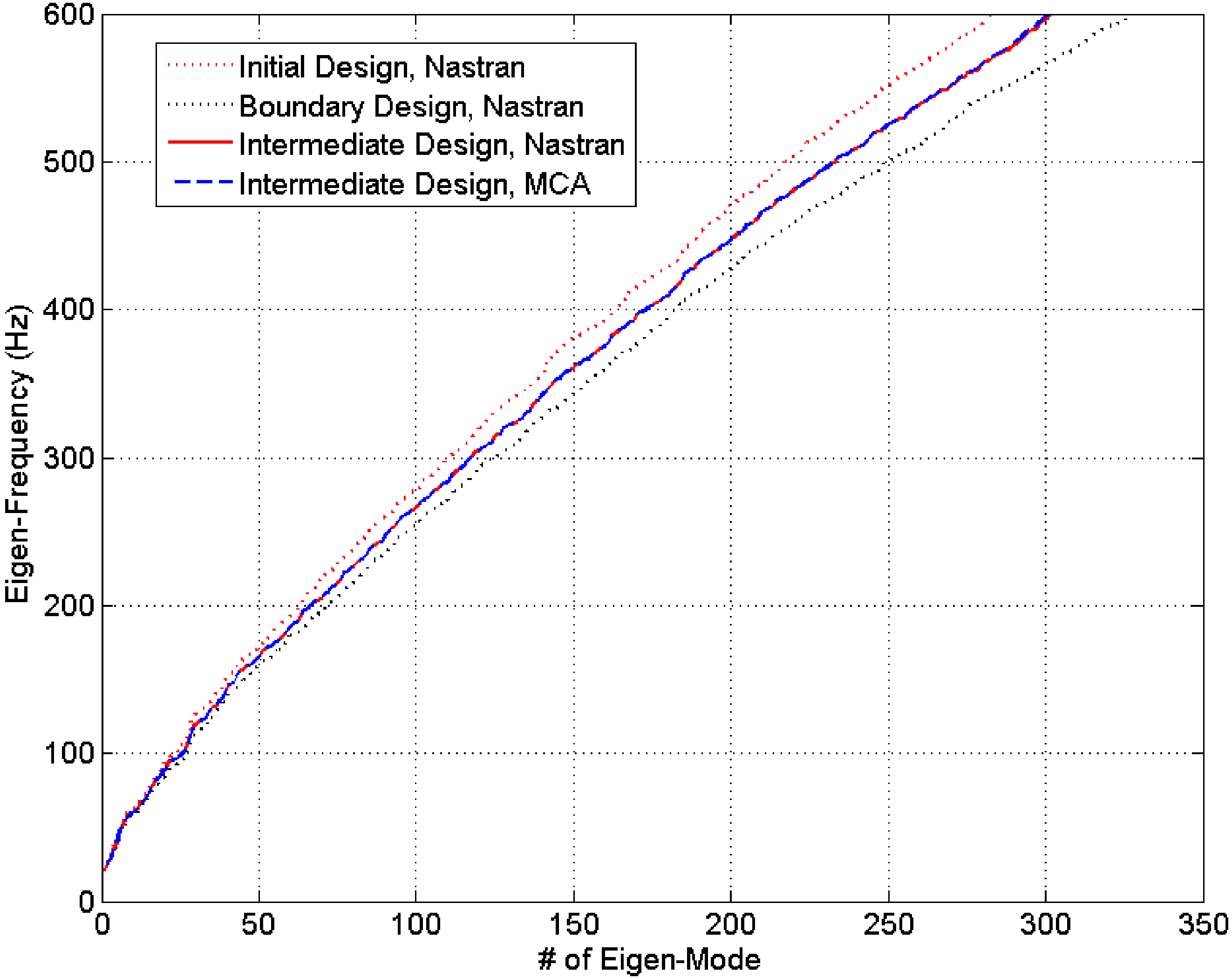
A Door Example



Same number of DOFs

- **150K DOFs**
- **Maximum L = 152 mm**
- **New Design L = 76 mm**

PROM and MCA for Shape Changes
A Door Example

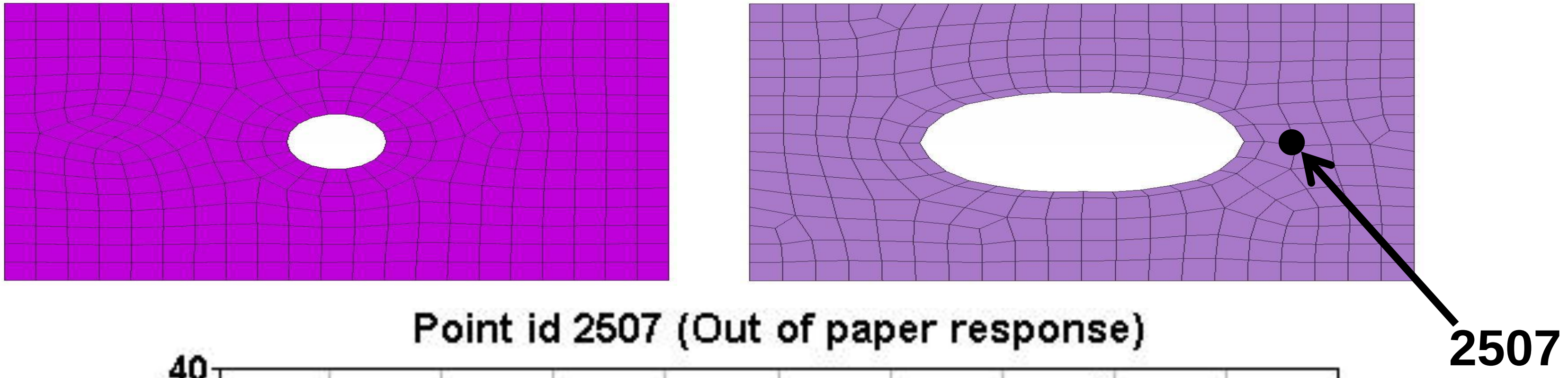


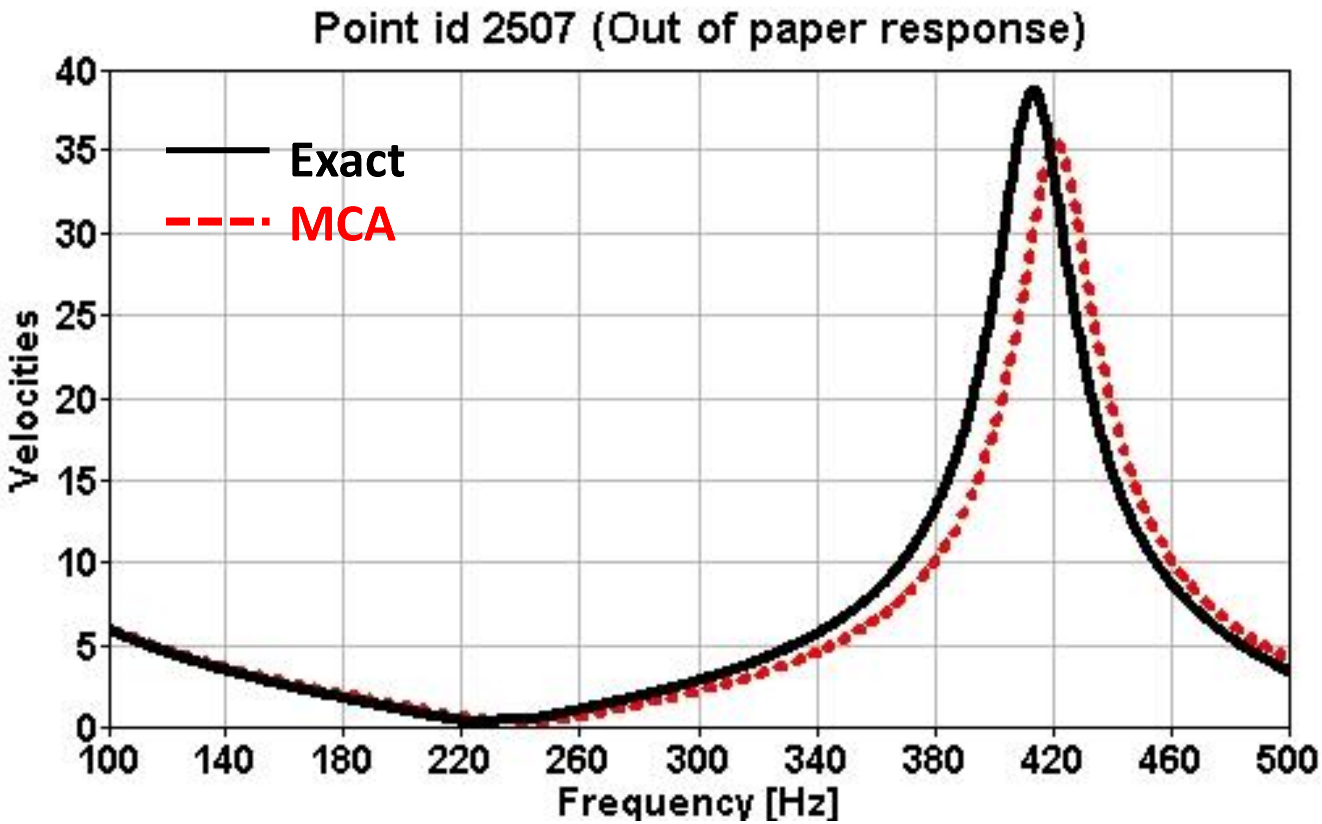
MCA Re-Analysis

Error less than 0.01%

MCA cost = ½ of PROM

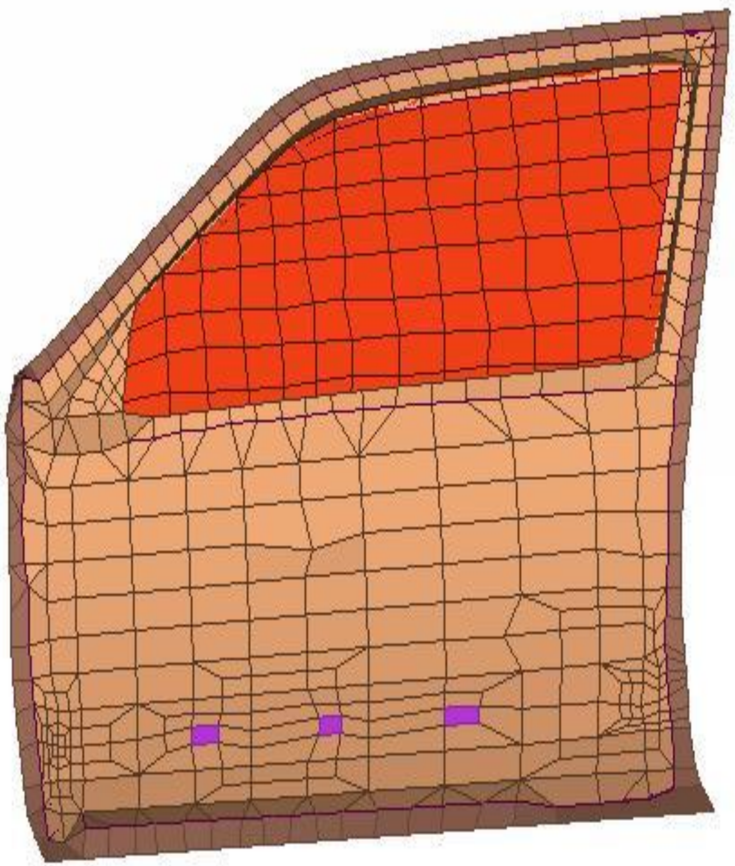
MCA for Shape Changes: Different DOFs



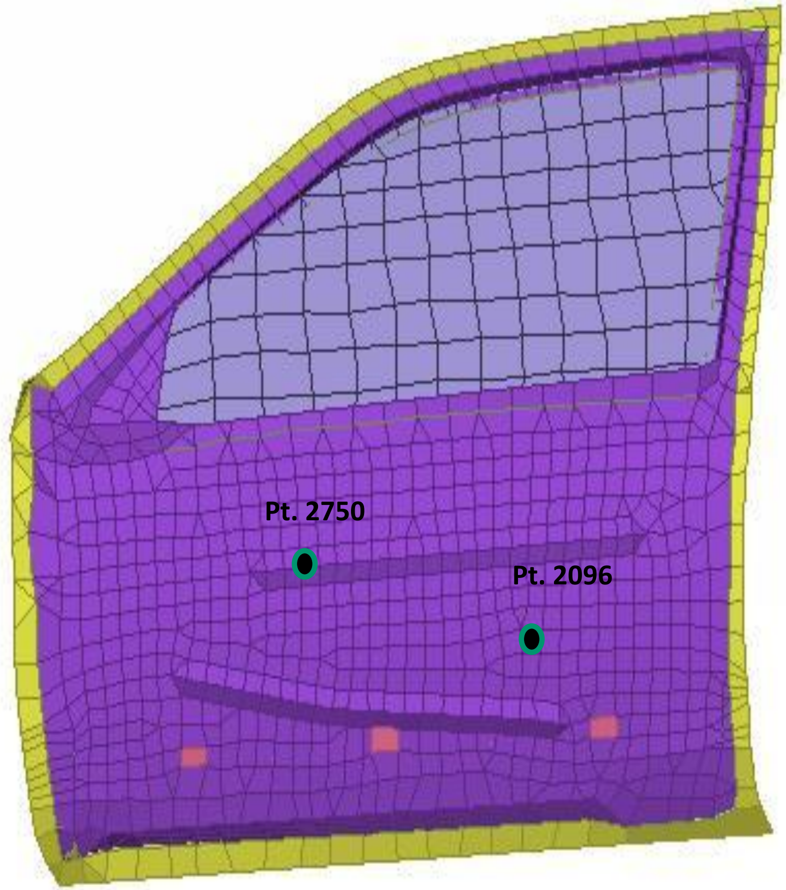


MCA for Shape Changes: Different DOFs

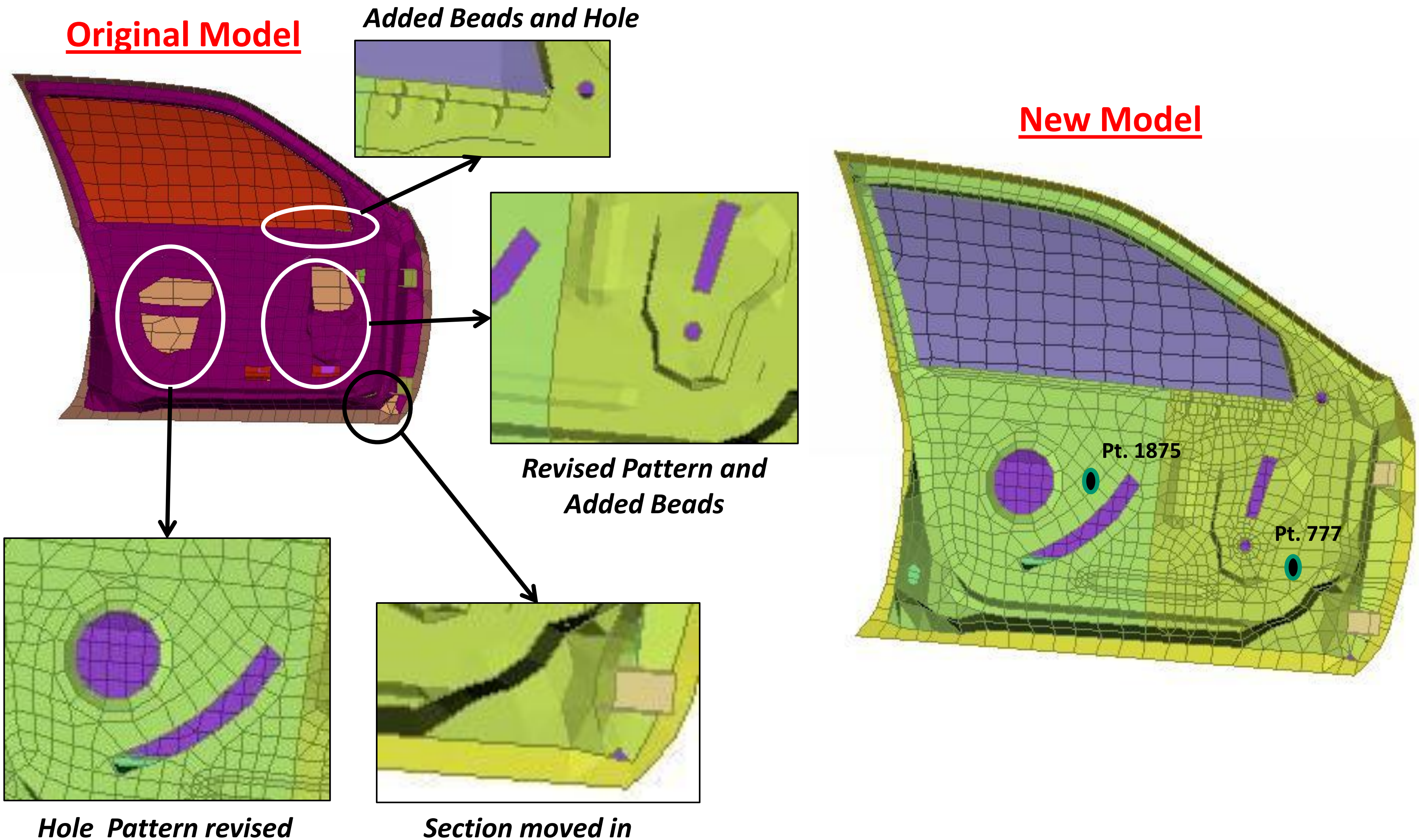
Original Model



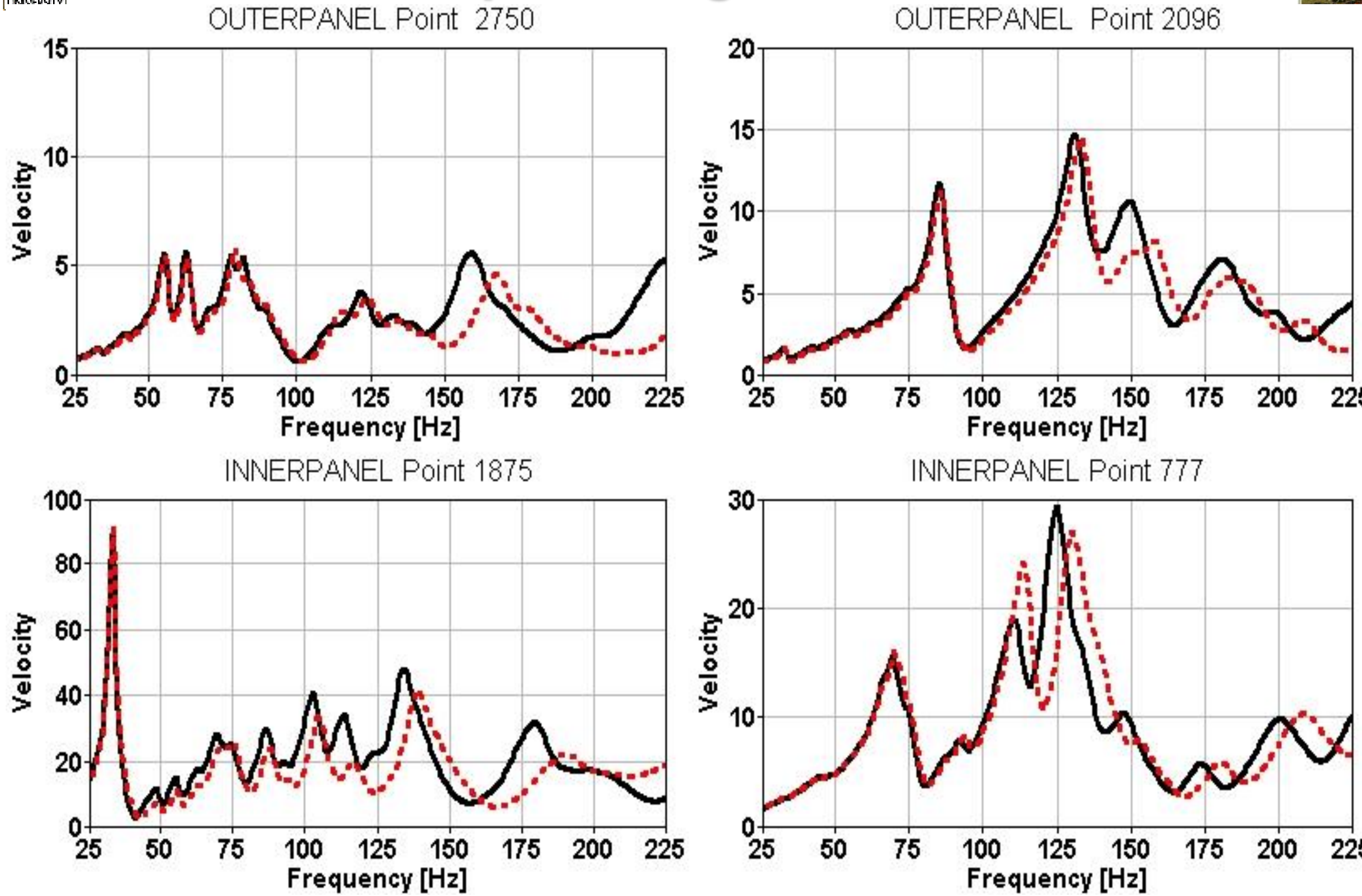
New Model



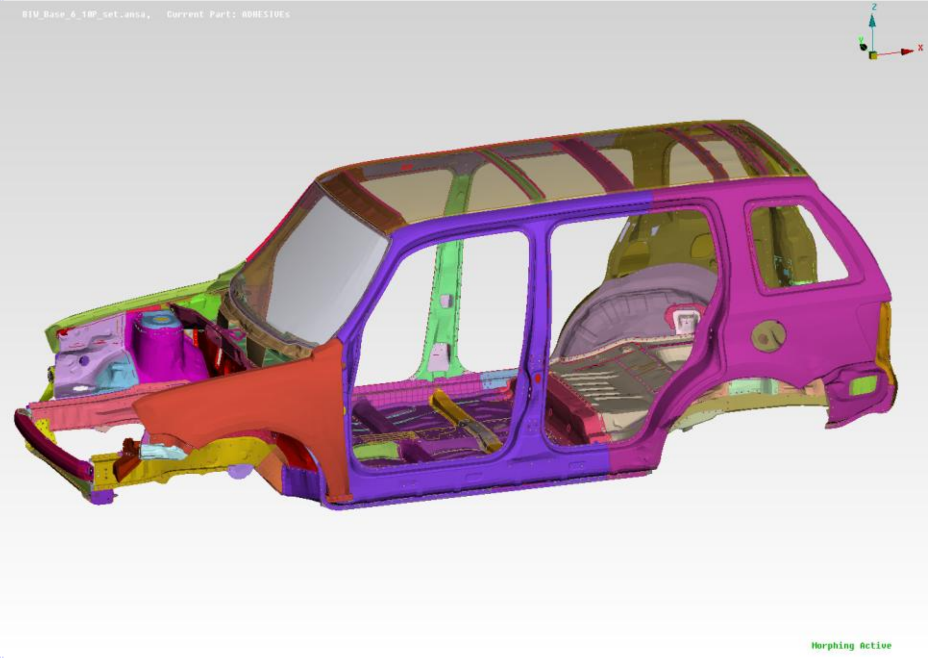
MCA for Shape Changes: Different DOFs



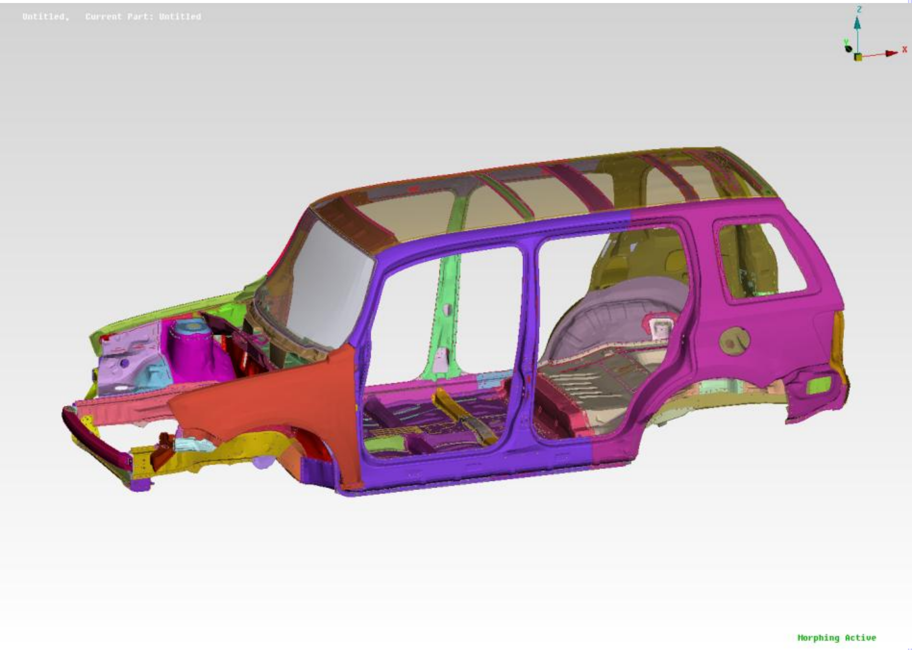
MCA for Shape Changes: Different DOFs



Baseline and Optimal Design

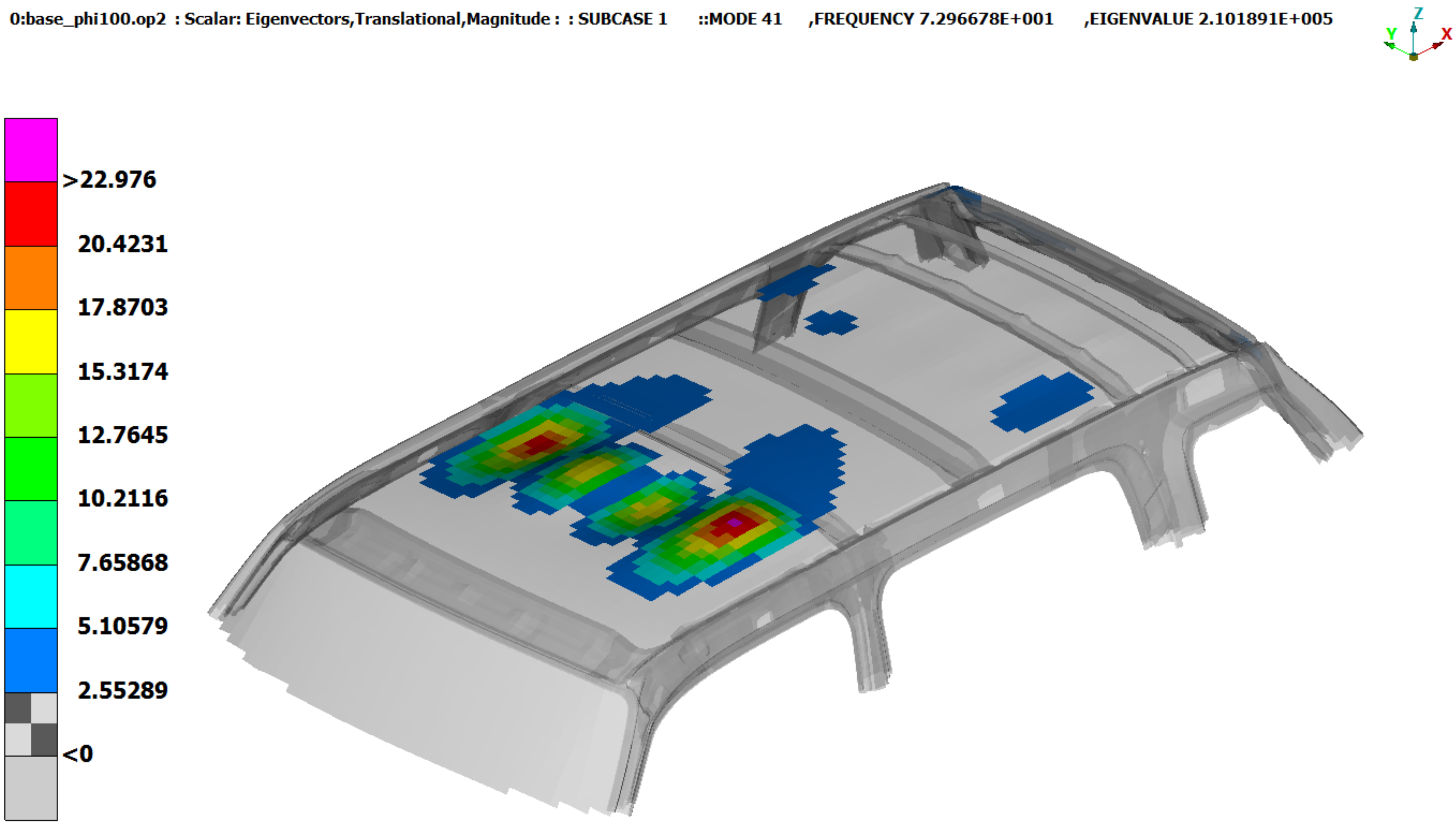


Baseline Design

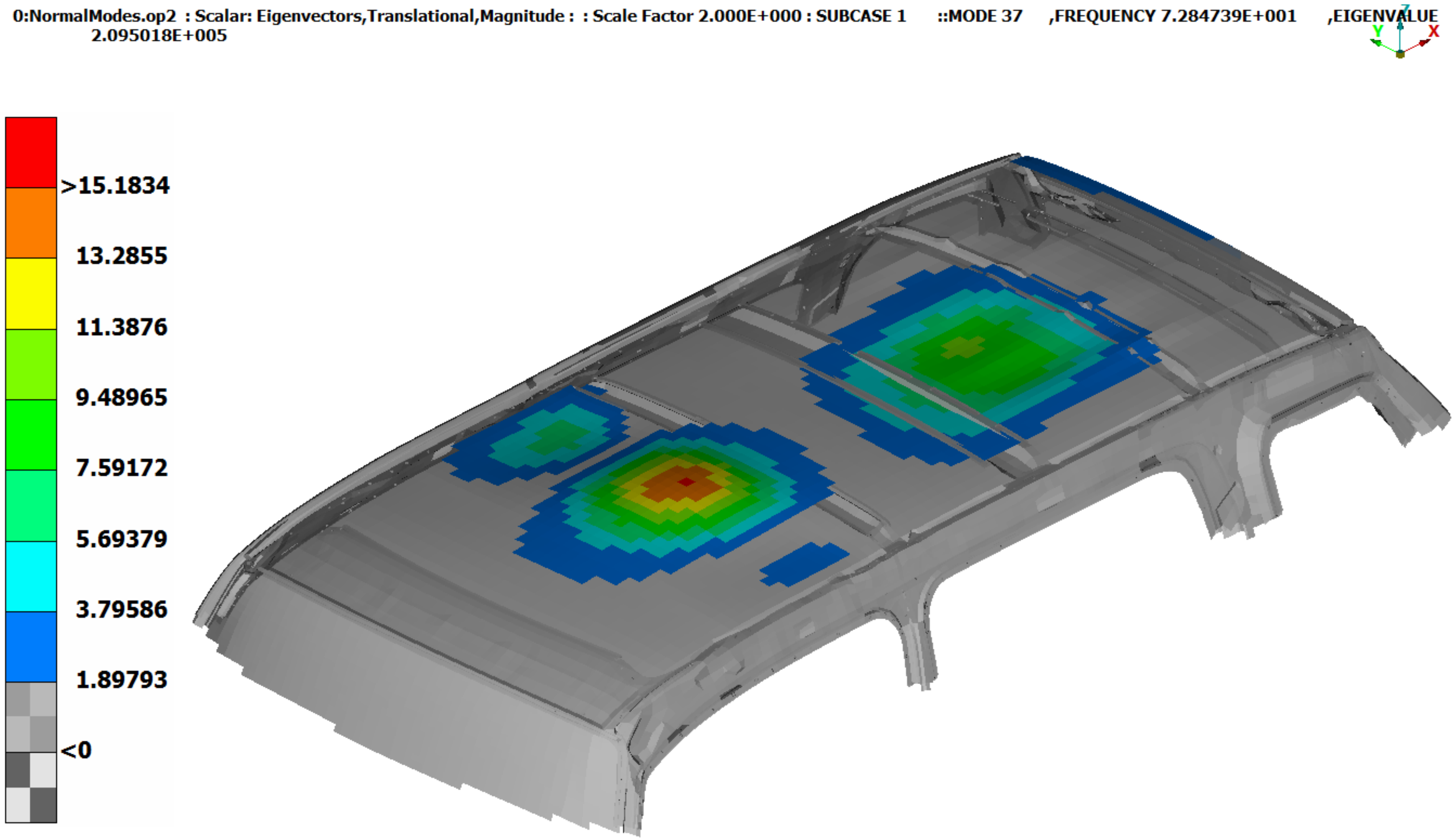


Optimal Design

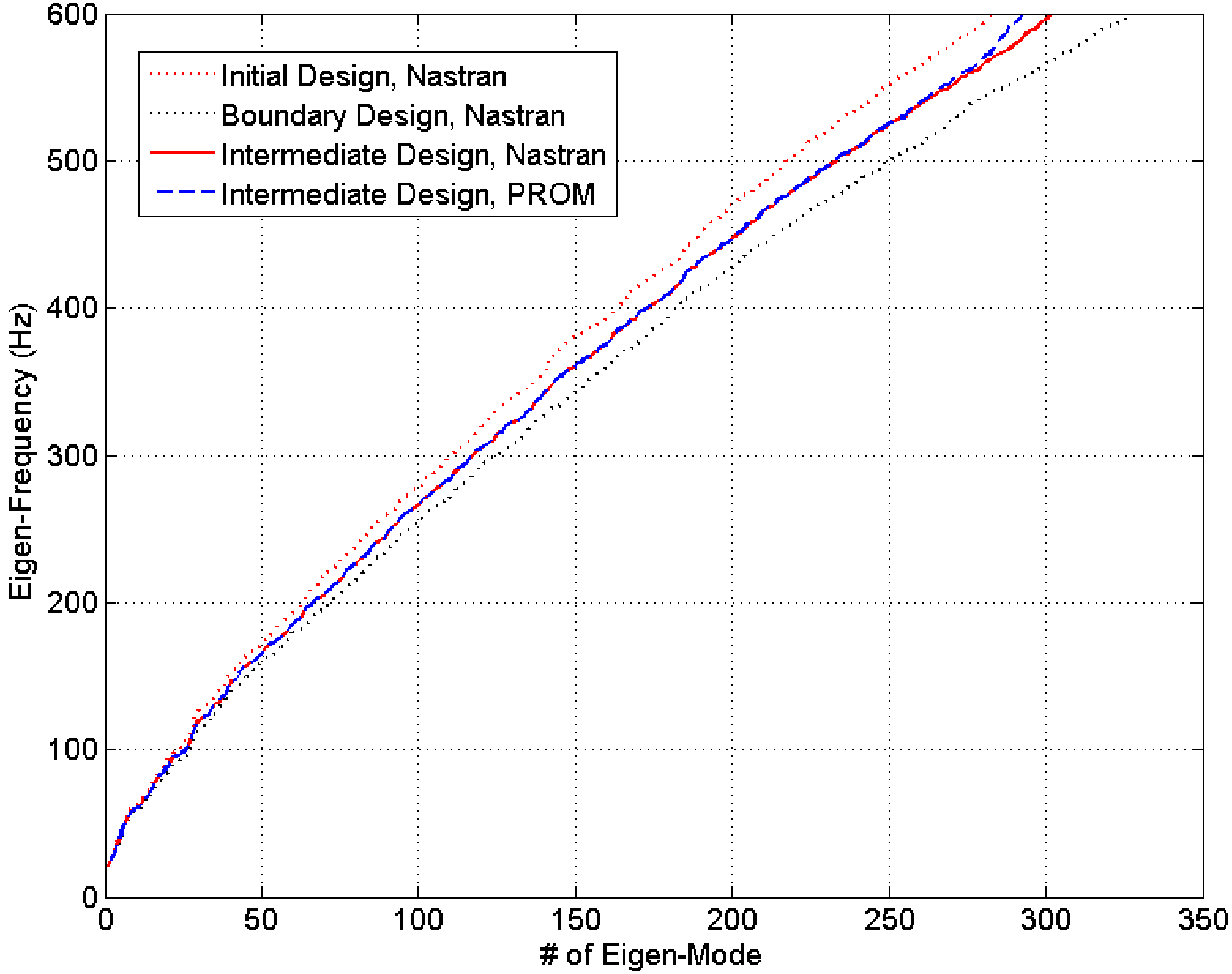
Roof Response: Baseline at 73 Hz



Roof Response: Optimal Design at 72.8 Hz



PROM and MCA for Shape Changes
A Door Example



PROM Re-Analysis

Error less than 0.05%