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# Adaptive FEM for Aerospace and Aeroacoustics Applications

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# About us

## What do we do?

- ▶ Develop the **open-source** turbulent flow solver **UNICORN**.
  - ▶ *FEniCS open-source project.*
  - ▶ *MPI-IO, parallel mesh-refinement, dynamic load-balancing.*
  - ▶ *Adjoint based mesh adaptivity.*
  - ▶ *Linear scalability up to 12,000 cores.*
- ▶ Study **turbulent flow** phenomena with several **applications**.
- ▶ 1 Professor, 1 senior researcher, 2 post-docs, 5 PhD students.

## Who am I?

- ▶ 4<sup>th</sup> year (of 5) **PhD candidate**.
- ▶ Use **UNICORN** to study **aerodynamics** and **aeroacoustics**.
  - ▶ *Separation, airframe noise (landing gear, slat-noise, etc) and duct-acoustics.*

# Collaboration with ANSA

## Workflow



# Collaboration with ANSA

## Workflow



Difficult to use for complex geometries...

## Collaboration with ANSA

### ANSA mesh generation



*From left to right...*

- ▶ 30P30N from NASA, benchmark workshops BANC-I and BANC-II.
- ▶ Gulfstream G550 nose landing gear, also BANC-I and BANC-II.
- ▶ DLR model airplane, High-Lift Prediction Workshop 2.

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# Adaptive FEM for turbulent flows

## General Galerkin (G2)

- ▶ **FEM** with piecewise linear approximation in space and time.
- ▶ Fully **unstructured meshes**.
- ▶ Time-resolved method where **numerical stabilization** accounts for unresolved scales.
- ▶ **Simple wall shear stress model** based on skin friction, slip velocity boundary condition, in the spirit of simpler models.<sup>1</sup>
- ▶ **Adaptive mesh refinement** with respect to output of interest using associated adjoint problem to estimate errors in output.

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<sup>1</sup>U. Schumann, *Subgrid scale model for finite difference simulations of turbulent flows in plane channels and annuli.*

# Adaptive FEM for turbulent flows

## Adjoint-based mesh refinement

For  $\hat{U} = (U, P)$  a weak solution,  $\hat{\phi} = (\varphi, \theta)$  a solution to a linearized adjoint problem, and  $M(\hat{U}) = ((\hat{U}, \hat{\psi}))$  a mean value output, with  $\hat{\psi}$  a weight function, we define the error estimate:

$$|M(\hat{u}) - M(\hat{U})| = |((\hat{u} - \hat{U}, \hat{\psi}))| \leq \sum_{K \in \mathcal{T}_n} \mathcal{E}_K,$$

with the error indicator:

$$\mathcal{E}_K \equiv \sum_{n=1}^N \left[ \int_{I_n} |R_1(\hat{U})|_K \cdot \omega_1 dt + \int_{I_n} |R_2(U)|_K \omega_2 dt + \int_{I_n} |SD_{\delta}^n(\hat{U}; \hat{\phi})_K| dt \right],$$

for each element  $K$  in the mesh  $\mathcal{T}_n$ , with stability weights  $\omega_i, i = 1, 2$ .

# Adaptive FEM for turbulent flows

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with the error indicator:

***error in  $M(\hat{u}) \equiv \underline{f}(\text{turbulence}, \text{adjoint solution})$***

for each element  $K$  in the mesh  $\mathcal{T}_n$ , with stability weights  $\omega_i, i = 1, 2$ .

# Adaptive Mesh Refinement

*How do we generate the mesh?*

## Adaptive algorithm

1. For the mesh  $\mathcal{T}_n$ : compute primal and adjoint problem.
2. Compute  $\mathcal{E}_K, K \in \mathcal{T}_n$ .
3. Mark 10% of the elements with highest “error indicator” for refinement.
4. Generate the refined mesh  $\mathcal{T}_{n+1}$ , and goto 1.

Example 30P30N high-lift wing:

Initial mesh: 1M cells.

Mesh after 7 adaptive refinements: 6.6M cells.

⇒ Compare, e.g., with Imamura et al, 16.3M points!<sup>2</sup>

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<sup>2</sup>Imamura, T., Murayama, M., Hirai, T., and Yamamoto, K., *Aeroacoustic Simulations around 30P30N, JAXA's Result,* Proceedings for BANC-II, 2012.



# Adaptive Mesh Refinement

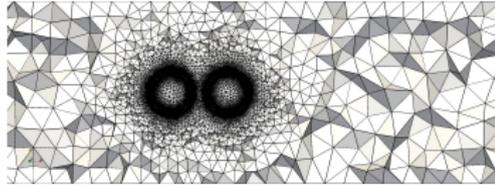
How to choose the refinement target  $M(\hat{u})$ ?

*It depends on the application...*

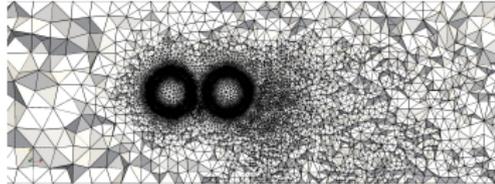
- ▶ For aerodynamics, **drag**, **lift** or **drag+lift**.
- ▶ For external aeroacoustics, **Lighthill's analogy**.
- ▶ Duct acoustics, **pressure drop**.
- ▶ ...

# Adaptive Mesh Refinement

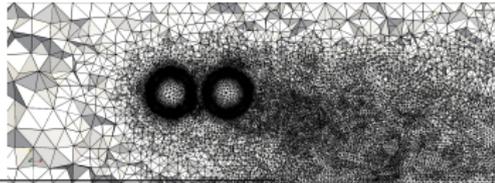
Initial mesh<sup>3</sup>:



3 refinements:



9 refinements:



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<sup>3</sup>Vilela de Abreu et al, Adaptive computation of aeroacoustic sources for a rudimentary landing gear using Lighthill's analogy, Proceedings for the 17th AIAA/CEAS Aeroacoustics Conference, 2011.



# Adaptive Mesh Refinement

What are the advantages of an adaptively generated mesh?

- ▶ Mesh captures the relevant flow features.
- ▶ No need for *ad hoc* meshing.
- ▶ No need for a “mesh study”<sup>4</sup>.
- ▶ Final mesh has “optimal” size.

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<sup>4</sup>A hierarchy of meshes is automatically generated by the adaptive algorithm and flow solutions are available for all meshes. Moreover, a stop criterion for the algorithm should be chosen to ensure “mesh convergence”.



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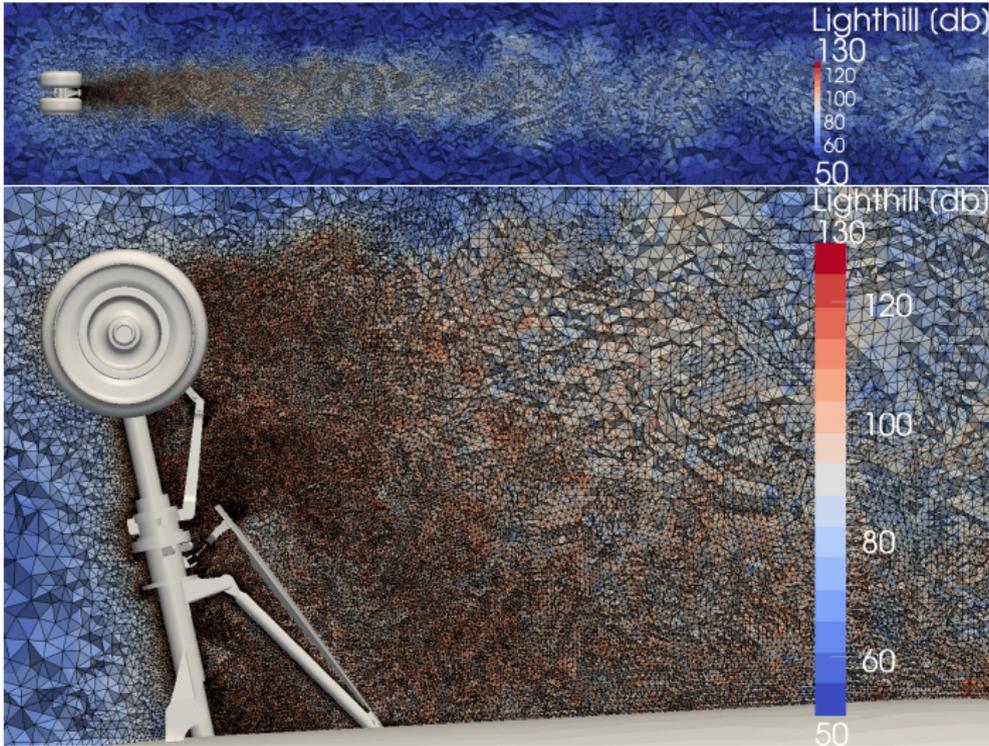
# Adaptive Mesh Refinement

*Mesh captures the relevant flow features...*



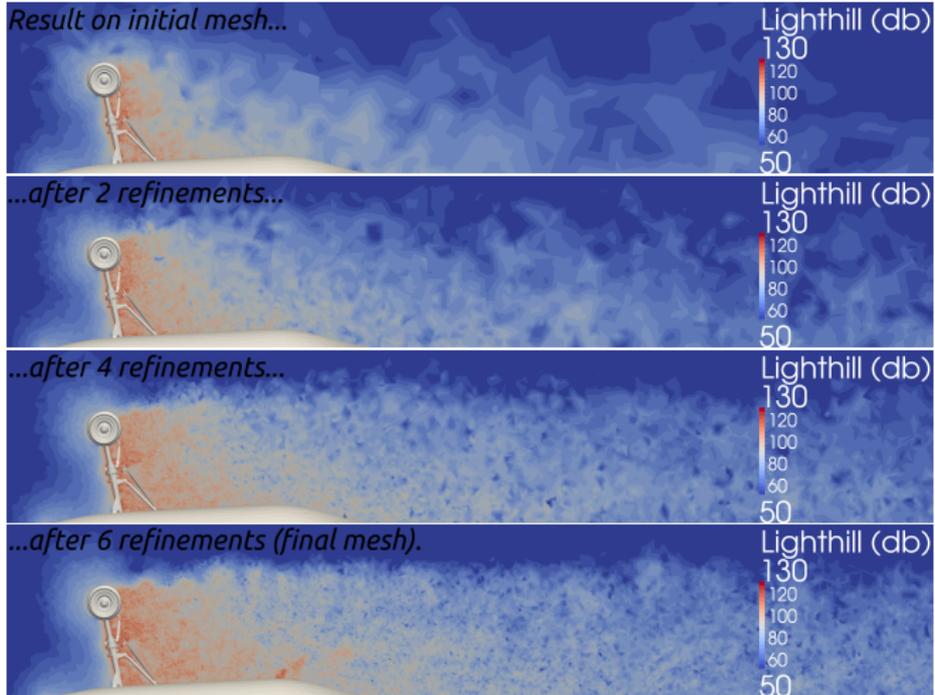
# Adaptive Mesh Refinement

*Mesh captures the relevant flow features...*

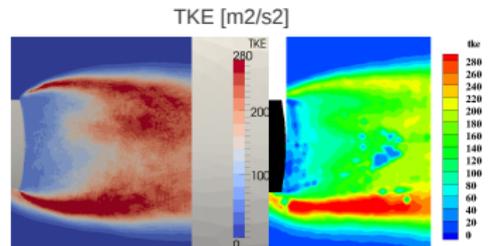
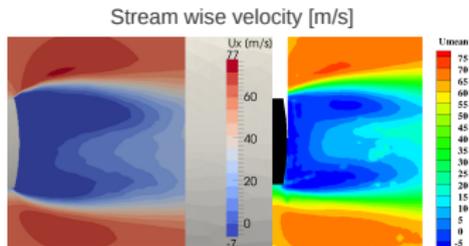
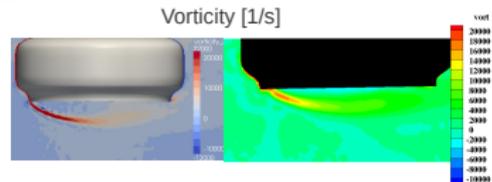
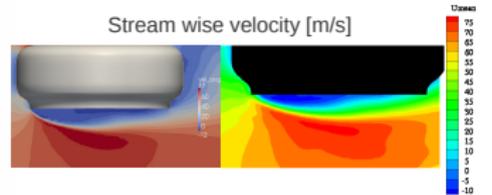
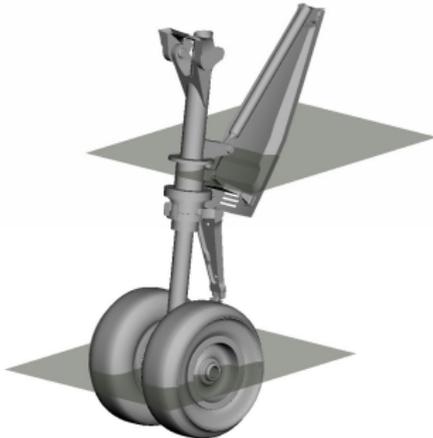


# Adaptive Mesh Refinement

*Solution on different meshes...*

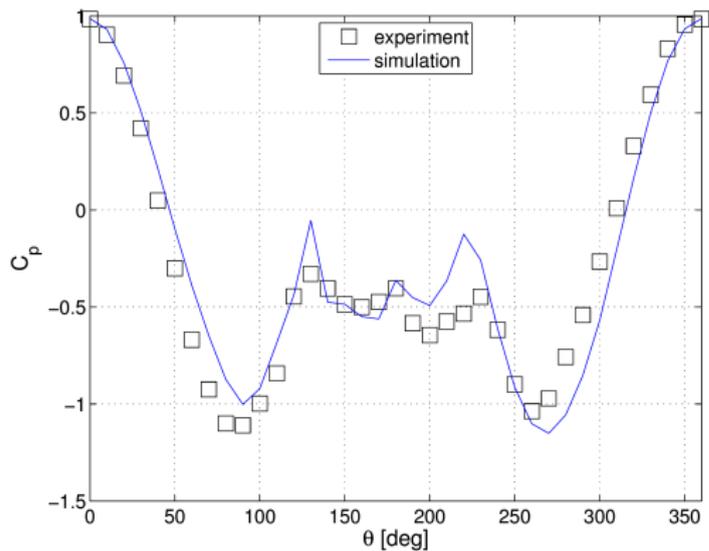


# Benchmark results, BANC-II



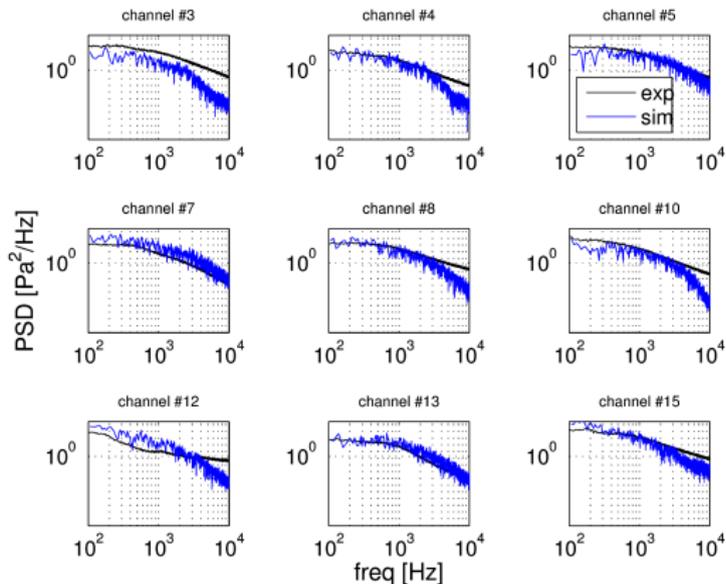
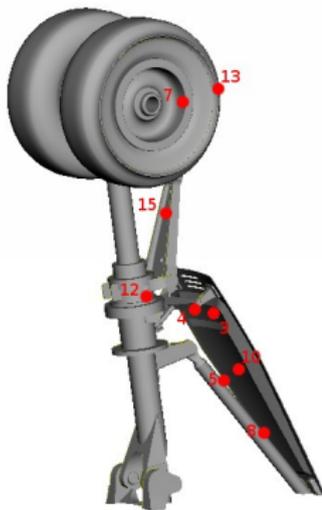
In *all* figures: left, *sim*; right, *exp*.

## Benchmark results, BANC-II



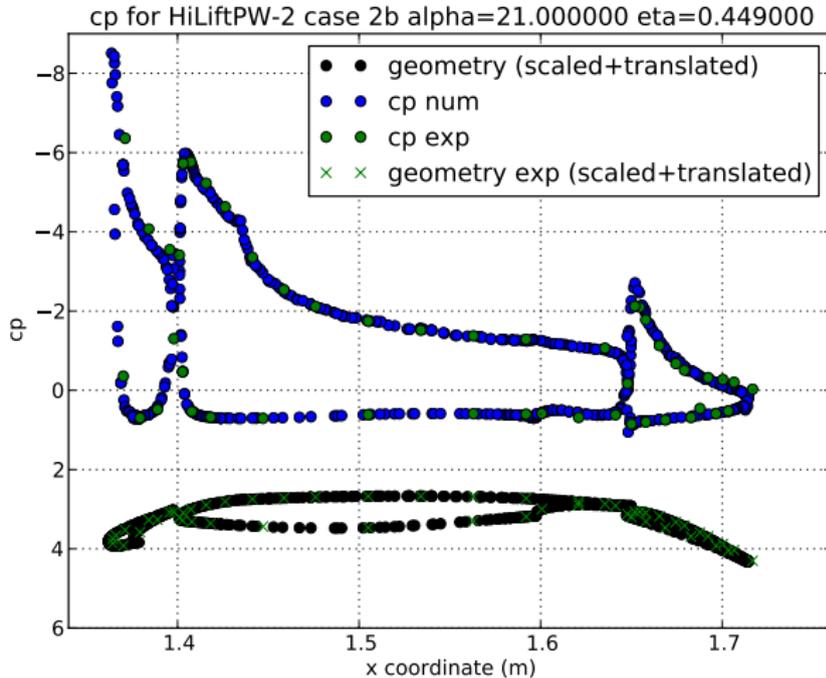
Mean static *pressure coefficient* distribution.

# Benchmark results, BANC-II



*Power Spectral Density **unsteady** pressure.*

# Benchmark results, HiLiPW-2



*Mean static pressure coefficient distribution.*



# Why ANSA?

## Enabling features

- ▶ Easy to clean-up geometries, even for new users.
- ▶ Batch mesh generation.
- ▶ Precise control of parameters (*e.g. leading edge curvature, growth rate, min-max cell sizes, quality*).
- ▶ High quality volume mesh (highly required in our framework for refinement).
- ▶ ...

⇒ *Very knowledgeable, efficient and helpful support! Thanks Vangelis!*



## Unicorn and DOLFIN, open source

<http://launchpad.net/unicorn>

## Acknowledgement

All initial meshes were generated with **ANSA** by Beta CAE Systems.

The code **Saaz** was used in “offline mode” for post-processing.<sup>5</sup>

Financial support from

- ▶ Swedish Foundation for Strategic Research
- ▶ European Research Council
- ▶ Swedish Research Council, Swedish Energy Agency

This work was performed on resources provided by the Swedish National Infrastructure for Computing (**SNIC**) at the Center for High-Performance Computing (**PDC**) at KTH.

<sup>5</sup>Alden King, Eric Arobone, Scott B. Baden and Sutanu Sarkar, *The Saaz Framework for Turbulent Flow Queries*, 2011.